

RVK/KW/13/6584/6589

Faculty of Engineering & Technology
Third Semester B.E. (Computer Technology/CBS)
(C.B.S.) Examination

APPLIED MATHEMATICS—III

Paper—III

Vaikunth

Time : Three Hours]

[Maximum Marks : 80

INSTRUCTIONS TO CANDIDATES

- (1) All questions are compulsory.
- (2) Due credit will be given to neatness and adequate dimensions.
- (3) Assume suitable data wherever necessary.
- (4) Use of non-programmable calculator is permitted.
- (5) Solve **SIX** questions as follows :
 - (i) Q. No. 1 OR Q. No. 2
 - (ii) Q. No. 3 OR Q. No. 4
 - (iii) Q. No. 5 OR Q. No. 6
 - (iv) Q. No. 7 OR Q. No. 8
 - (v) Q. No. 9 OR Q. No. 10
 - (vi) Q. No. 11 OR Q. No. 12

- 1 (a) Find Laplace Transform of $\frac{\sin^2 t}{t}$ and hence

evaluate $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt$. 6

- (b) Find $L^{-1} \left\{ \frac{1}{(s+1)(s^2+1)} \right\}$ by using Convolution theorem. 6

OR

2. (a) Find Laplace Transform of periodic function :

$$f(t) = \begin{cases} \cos wt, & 0 < t < \pi/w \\ 0, & \pi/w < t < 2\pi/w. \end{cases} \quad 6$$

- (b) Solve the differential equation using Laplace Transform method :

$$(D^2 + 2D + 5)y = e^{-t} \sin t, \text{ given}$$

$$y(0) = 0, y'(0) = 1 \text{ where } D = \frac{d}{dt}. \quad 6$$

3. (a) Find Fourier series for $f(x) = x - x^2$ in interval $-1 < x < 1$. 6

- (b) Find the Fourier sine transform of :

$$f(x) = \frac{e^{-ax}}{x}, a > 0. \quad 6$$

OR

4. (a) Find Fourier series for

$$f(x) = \begin{cases} \pi + x, & -\pi < x \leq 0 \\ \pi - x, & 0 \leq x < \pi \end{cases}$$

and hence show that :

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \quad 6$$

- (b) Using Fourier integral, show that :

$$\int_0^{\infty} \frac{w \sin(xw)}{1+w^2} dw = \frac{\pi}{2} e^{-x}, x > 0. \quad 6$$

5. (a) Find Z-Transform of $\sin n\theta$ and $\cos n\theta$. 6

- (b) Find inverse Z-Transform of

$$\frac{z^2 + z}{(z-1)(z^2 + 1)} \quad 6$$

OR

6. (a) If $z \{f(n)\} = F(z)$, then show that :

$$z \left\{ \frac{f(n)}{n+k} \right\} = z^k \int_z^{\infty} \frac{F(z)}{z^{k+1}} dz \text{ and}$$

hence find z-Transform of $\frac{1}{n+1}$. 6

11. (a) Let X be the random variable giving the number of heads in three tosses of a fair coin.

Find (i) probability function $f(x)$

(ii) distribution function $F(x)$. 6

- (b) Can the function, $F(x) = \begin{cases} c(1-x^2), & 0 \leq x \leq 1 \\ 0 & , \text{ otherwise} \end{cases}$ be a distribution function? Explain. 6

OR

12. (a) Find mean, variance and moment generating function for exponential distribution

$$f(x) = \begin{cases} \alpha e^{-\alpha x}, & x > 0 \\ 0 & , x \leq 0 \end{cases} \quad 6$$

- (b) Let X and Y be random variables having joint density function $f(x, y) = \begin{cases} e^{-(x+y)}, & x \geq 0, y \geq 0 \\ 0 & \text{ otherwise} \end{cases}$

Find (i) $\text{Var}(x)$

(ii) σ_x

(iii) ρ . 6