

SPK/KW/12/6437

Faculty of Engineering & Technology

First Semester B.E. Examination

APPLIED MATHEMATICS—I

Paper—I

Time—Three Hours]

[Maximum Marks—80

INSTRUCTIONS TO CANDIDATES

(1) Solve **SIX** questions as follows :

Que. 01 OR Que. 02, Que. 03 OR Que. 04,

Que. 05 OR Que. 06, Que. 07 OR Que. 08,

Que. 09 OR Que. 10, Que. 11 OR Que. 12.

(2) Use of non-programmable calculator is permitted.

1. (a) If $y = e^{\tan^{-1} x}$, prove that :

$$(1 + x^2) y_{n+2} + [2(n + 1)x - 1] y_{n+1} + n(n + 1) y_n = 0.$$

6

(b) Evaluate :

$$(i) \lim_{x \rightarrow 0} \left[\frac{1}{x} - \cot x \right]$$

3

(ii) $\lim_{x \rightarrow 0} (1 + \sin x)^{\cot x}$

OR

2. (a) For the curve $(a + x)y = ax$, if P is the radius of curvature at a point (x, y) , show that

$$\left(\frac{2P}{a}\right)^{2.1} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2 \quad 6$$

- (b) Using Taylor's series find the value of $\cos 64^\circ$ correct upto four decimal places 6

3. (a) Find the value of n , so that the equation $v = r^n (3 \cos \theta - 1)$ satisfies the relation :

$$\frac{\partial}{\partial r} \left(r^2 \frac{\partial v}{\partial r} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial v}{\partial \theta} \right) = 0. \quad 6$$

- (b) If $u = f(r, s, t)$ and $r = \frac{x}{y}$, $s = \frac{y}{z}$, $t = \frac{z}{x}$, find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$. 6

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(c) If $u = \log \left[\frac{x^4 - y^4}{x - y} \right]$

prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -3. \quad 6$$

OR

4. (a) Given $u = 4x^2 + 9y^2 + 16z^2$, $v = 12xy + 16xz + 24yz$, $w = 2x + 3y + 4z$. Are u, v, w functionally related? If so, find the relation between them. 6
- (b) Expand $x^3y + 3y = 2$ in power of $(x - 1)$ and $(y + 2)$ 6
- (c) A rectangular box, open at the top is to have a volume of 32 C.C. Find the dimensions of the box requiring least material for its construction. 6

5. (a) Find the inverse of matrix

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \text{ by Partitioning} \quad 7$$

(b) Test the consistency and solve

$$5x + 3y + 7z = 4,$$

$$3x + 26y + 2z = 9 \text{ and}$$

$$7x + 2y + 10z = 5$$

5

OR

6. (a) By adjoint method solve the system of equations

$$5x + 3y + 3z = 48,$$

$$2x + 6y - 3z = 18,$$

$$8x - 3y + 2z = 21$$

6

(b) Find the Rank of the matrix

$$\begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix}$$

6

7. (a) $\frac{dy}{dx} = \frac{y+1}{(y+2)e^y - x}$

4

(b) $(2xy + y - \tan y) dx + (x^2 - x \tan y + \sec^2 y) dy = 0$

4

(c) $\frac{dy}{dx} + x \sin 2y = x^{-1} \cos^2 y$

4

OR

8 (a) Solve

$$p^2 + 2py \cot x = y^2$$

3

(b) Solve

$$y = 2(px + y^2p)$$

3

(c) The equation of electromotive force in terms of current i for an electrical circuit having resistance R and condenser of capacity C in series is

$$E = Ri + \int \frac{i}{C} dt. \text{ Find the current } i \text{ at any time}$$

t when $E = E_m \sin \omega t$.

6

9 (a) Solve

$$\frac{d^4 y}{dx^4} - y = \cos x \cosh x.$$

6

(b) Solve by method of variation of parameters

$$\frac{d^2 y}{dx^2} - y = \frac{2}{1+e^x}$$

6

(c) $x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 5y = x^2 \sin(\log x)$

6

OR

10 (a) Solve the simultaneous differential equations

$$\frac{dx}{dt} + y = \sin t \quad \frac{dy}{dt} + x = \cos t,$$

given that $x = 2$ and $y = 0$ when $t = 0$. 6

(b) Solve $\frac{d^2x}{dx^2} = \sec^2 y \tan y$, given that $y = 0$ and

$$\frac{dx}{dx} = 1 \text{ when } x = 0. \quad 5$$

(c) A spring for which stiffness $K = 700 \text{ N/m}$ hangs in a vertical position with its upper end fixed. A mass of 7 kg is attached to the lower end. After coming to rest, the mass is pulled down 0.05 m and released. Discuss the resulting motion of the mass, neglecting air resistance. 7

11. (a) Find all the values of $(16)^{1/4}$ in (a + ib) form. 4

(b) If $\cos(\theta + i\phi) = \cos \alpha + i \sin \alpha$, prove that :
 $\cos 2\theta + \cosh 2\phi = 2$. 4

OR

12. (a) Express $\sin 7\theta$ in terms of powers of $\cos \theta$ and $\sin \theta$. 4

(b) If $\tan(\theta + i\phi) = \cos \alpha + i \sin \alpha$, prove that

$$\theta = \frac{n\pi}{2} \text{ or } \frac{\pi}{4}$$

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