Faculty of Engineering & Technology

Third Semester B.E. (Computer Technology)/C.S.E.

(C.B.S.) Examination

APPLIED MATHEMATICS—III

Time—Three Hours] [Maximum Marks—80

INSTRUCTIONS TO CANDIDATES

(1) All questions carry marks as indicated.

(2) Solve **SIX** questions as follows :

Q.No. **1** OR Q.No. **2**

Q.No. **3** OR Q.No. **4**

Q.No. **5** OR Q.No. **6**

Q.No. **7** OR Q.No. **8**

Q.No. **9** OR Q.No. **10**

Q.No. **11** OR Q.No. **12**.

(3) Use of Non-programmable calculator is permitted.

(4) Assume suitable data wherever necessary.

1. (a) If $L\{f(t)\} = F_{(s)}$, then show that :

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F_{(s)}\,ds$$ and also find $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$. 6

(b) Find $L^{-1}\left[\frac{1}{s(s^2+1)}\right]$ by Convolution Theorem. 6

OR

2. (a) Express $f(t) = \begin{cases} \sin t & , \quad 0 < t < \pi \\ \sin 2t & , \quad \pi < t < 2\pi \\ \sin 3t & , \quad t > 2\pi \end{cases}$

in terms of unit step function and find its Laplace transform. 6

(b) Solve $(D^2 + 2D + 5)\,Y = e^{-t}\sin t$

given $Y(0) = 0$, $Y'(0) = 1$, where $D = \frac{d}{dt}$. 6

3. (a) Find Fourier series for :

$$f(x) = \begin{cases} \pi + x & ; \quad -\pi < x \le 0 \\ \pi - x & ; \quad 0 \le x < \pi \end{cases}$$

and hence show that :

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$$ 6

(b) Using Fourier integral, show that :

$$\int_0^{\infty} \frac{\sin \pi\lambda \, \sin \lambda x}{1 - \lambda^2}\, d\lambda = \begin{cases} \left(\frac{\pi}{2}\right)\sin x & , \quad 0 \le x \le \pi \\ 0 & , \quad x > \pi \end{cases}$$ 6

OR

4. (a) Find half range sine series for $f(x) = (\pi x - x^2)$ in the interval $(0, \pi)$. 6

(b) Find the Fourier Sine Transform of $e^{-|x|}$ and hence show that :

$$\int_0^{\infty} \frac{x \sin mx}{1 + x^2}\, dx = \frac{\pi}{2} e^{-m} \; ; \; m > 0.$$ 6

5. (a) Find Z-transform of $\sin n\theta$ and $\cos n\theta$. 6

(b) Find inverse Z-transform of $\frac{z^4}{(z-a)^4}$. 6

OR

6. (a) If $z\{f(n)\} = F(z)$, then show that :

$$z\left[\frac{f(n)}{n+k}\right] = z^{k}\int_{z}^{\infty}\frac{F(z)}{z^{k+1}}dz$$

and hence find Z-transform of $\frac{1}{n+1}$. 6

(b) Using Z-transform method, solve :

$u_{n+2} + 4u_{n+1} + 3u_n = 2^n$, given $u_0 = 0$, $u_1 = 1$. 6

7. (a) Prove that $u = y^3 - 3x^2y$ is harmonic function. Find its conjugate and the corresponding analytic function f(z) in terms of z. 7

(b) Evaluate $\oint \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}dz$ by Cauchy Residue theorem. 7

OR

8. (a) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent's series valid for :

(i) $1 < |z| < 3$,

(ii) $|z| > 3$. 7

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(b) Evaluate $\int_{0}^{2\pi}\frac{1}{5-4\sin\theta}d\theta$ by Contour Integration. 7

9. (a) Find the value of $M^2 - 3M + I$ and verify the result by using Sylvester Theorem, where :

$$M = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}.$$ 6

(b) Find the Modal Matrix B and verify $B^{-1}AB$ a diagonal matrix if :

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}.$$ 6

(c) Verify Cayley Hamilton Theorem for the matrix :

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & -1 \end{bmatrix}.$$ 6

OR

10. (a) Determine the largest eigen value and corresponding eigen vector of the matrix :

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$ 6

(b) Solve by Matrix Method $\frac{d^2x}{dt^2} + 4x = 0$ given $x(0) = 1$, $x'(0) = 0$. 6

(c) Are the following vectors linearly dependent ? If so, find relation between them : $x_1 = [1 \;\; 2 \;\; 4]$, $x_2 = [2 \;\; -1 \;\; 3]$, $x_3 = [0 \;\; 1 \;\; 2]$, $x_4 = [-3 \;\; 7 \;\; 2]$. 6

11. (a) In a bolt factory, Machines A, B and C manufacture respectively 25%, 35% and 40%. Of their total output 5%, 4%, 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by Machine A, B or C ? 6

(b) Let x be the random variable giving the number of heads in three tosses of a fair coin. Find :

(i) Probability function f(x)

(ii) Distribution function f(x). 6

OR

12 (a) Find the moment generating function of a random variable having density function

$$f(x) = \begin{cases} 2e^{-2x} & , \; x \geq 0 \\ 0 & , \; \text{otherwise} \end{cases}$$

and determine first four moments about origin. 6

(b) Let x and y be random variables having joint density function :

$$f(x,y) = \begin{cases} e^{-(x+y)} & , \; x \geq 0, y \geq 0 \\ 0 & , \; \text{otherwise} \end{cases}$$

Find : (i) var (x), (ii) σ_x. 6