III Sem B.E. CIVIL (New Course)

WINTER 2014

APPLIED MATHEMATICS - III

1. Expand = $f(x) = \sqrt{1-\cos x}$, $0 x < 2\pi$ in a Fourier Series.

Hence show that $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots\infty = \frac{1}{2}$. (7)

OR

2. If $f(x) = 2x - x^2$, expand f(x) as a Fourier series in the interval (0, 3) and hence deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots\infty = \frac{\pi}{12}.$$ (7)

3. (a) Solve $(x^2 - y^2 - z^2)\,p + 2xyq = 2xz$ (5)

(b) Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$ (7)

(c) Using the method of separation of variables, solve

$$\frac{3\partial u}{\partial x} + \frac{2\partial u}{\partial y} = 0,\ u(x,0) = 4e^{-x}.$$ (6)

OR

4. (a) A tightly stretched string with fixed end point x = 0 and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement y(x, t). (10)

(b) Find the vibration u(x, y, t) of a rectangular membrane $(0 < x < a, 0 < y < b)$ whose boundary is fixed given that it starts from rest and $u(x, y, 0) = hxy(a - x)(b - y)$. (8)

5. Find the curve passing through the points (x_1, y_1) and (x_2, y_2) which when rotated about the x-axis, gives a minimum surface area. (7)

OR

6. Find the plane curve of fixed perimeter and maximum area. (7)

7. (a) Are the vectors $x_1 = (1, 3, 4, 2)$, $x_2 = (3, -5, 2, 2)$ and $x_3 = (2, -1, 3, 2)$ linearly dependent ? If so express one of these as a linear combination of the others. (5)

(b) Verify Cayley - Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$ and find its inverse. Also express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A. (7)

(c) Use Sylvester's theorem to show that

$$2 \sin A = \{\sin 2\} A, \text{ where } A = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$$

OR

8. (a) Find the modal matrix corresponding to matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & 4 & 3 \end{bmatrix}$$

(b) Use matrix method to solve the differential equation.

$$\frac{d^2y}{dx^2} - \frac{5dy}{dx} + 6y = 0, \; y_{(0)} = 1, \; y^1_{(0)} = 2.$$ (6)

(c) Reduce the quadratic form

$3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form. (6)

9. (a) Find by Newton-Raphson method, the real root of $3x - \cos x - 1 = 0$ correct to four places of decimal. (6)

(b) Apply Crout's method to solve the equations : (6)
$3x + 2y + 7z = 4$, $2x + 3y + z = 5$, $3x + 4y + z = 7$.

(c) Using Runge - Kutta method of 4th order, find y for $x = 0\text{-}1$ & $0\text{-}2$, given that :

$$\frac{dy}{dx} = xy + y^2, y(0) = 1.$$

(6)

OR

10. (a) Find a real root of the equation $x \log_{10}x = 1.2$ by Regula - falsi method correct to four decimal places. (6)

(b) Apply Gauss-Seidal iteration method to solve the equations :

$3x + 20y - z = -18$

$2x - 3y + 20z = 25$

$20x + y - 2z = 17.$ (6)

(c) Apply Milne's Predictor - Corrector method, to find a solution of the differential equation $y' = x - y^2$ in the range $0 \le x \le 1$ for the boundary conditions $y = 0$ at $x = 0$. (6)

11. A company manufactures two types of cloth, using three different colours of wool, one yard length of type a cloth requires 4 m. of red wool, 5 m of green wool and 3 m of yellow wool. One yard length of type B cloth requires 5 m of red wool, 2 m of green wool and 8 m of yellow wool. The wool available for manufacture is 1000 m of red wool, 1000 m of green wool and 1200 m of yellow wool. The manufacturer can make a profit of Rs. 5 on one yard of type A cloth and Rs. 3 on one yard of type B cloth.

(a) Formulate the problems as a standard L.P.P. (6)

(b) Find product mix that would give maximum profit by graphical technique. (6)

OR

12. Solve the following L.P.P. by simplex method.

Maximize $Z = 5x_1 + 3x_2$

Subject to
$x_1 + x_2 \le 2$

$5x_1 + 2x_2 \le 10$

$3x_1 + 8x_2 \le 12$

$x_1, x_2 \ge 0.$ (12)

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