III Sem B.E. CIVIL (New Course)

SUMMER 2015

APPLIED MATHEMATICS - III

1. Obtain Fourier series for :

$$f(x) = \begin{cases} \pi x & , 0 \le x \le 1 \\ \pi(2-x) & , 1 \le x \le 2 \end{cases}$$

Hence, show that: $\frac{\pi x}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...$ (7)

OR

2. Find the half range (i) sine (ii) cosine Fourier series for the function $f(x) = x^2$ in the range $0 \le x \le \pi$. (7)

3. (a) Solve $x^2(y - z)\,p + y^2(z - x)\,q = z^2(x - y)$. (6)

(b) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial yz} = y \cos x.3$ (6)

(c) Solve $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = u$, given that $u(x,0) = 3e^{-5x} + 2e^{-3x}$ (6)

OR

4. (a) A tightly stretched string with fixed end points $x = 0, x = l$, is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $Xx(l - x)$. Find the displacement of the string at any distance from one end at any time t. (8)

(b) Solve $z(p - q) = z^2 + (x + y)^2$. (5)

(c) $\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x \partial y^2} + 6\frac{\partial^3 z}{\partial y^3} = \sin(x+2y)$.

5. Find the curve passing through the point (x_1, y_1) and (x2, y2) which when rotated about x-axis gives minimum surface area. (7)

OR

6. Prove that the sphere is the solid of revolution which, for given surface area, has maximum volume. (7)

7. (a) Find whether the following set of vectors are linearly dependent or otherwise, find the relation between them :

$x_1 = (1, 2, 1), x_2 = (4, 1, 2)$

$x_3 = (6, 5, 4), x_4 = (-3, 8, 1).$ (6)

(b) Find the latent roots and latent vectors and modal matrix B for the matrix :

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & 6 \\ -1 & -2 & 0 \end{bmatrix}$$ (6)

(c) Verify Cayley-Hamilton Theorem and express $A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$ as a linear polynomial in

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$ (6)

OR

8. (a) Using Sylvester Theorem, show that : $\sin^2 A + \cos^2 A = I$;

if $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$ (6)

(b) Solve $\frac{d^2y}{dt^2} 3 \frac{dy}{dt} 10y = 0$, given $y(0) = 3$,

$y'(0) = 15$ by Matrix Method. (6)

(c) Diagonalize by orthogonal transformation :

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$ (6)

9. (a) Using Rejgula-Falsi Method, find the root of the equation tan x + tan hx = 1, correct upto three places of decimal. (6)

(b) Solve by Gauss-Seidal Method :

$x + 7y - 3z = -22$

$5x - 2y + 3z = 18$

$2x - y + 6z = 22$ (6)

(c) Solve by using Euler's Modified Method :

$$\frac{dy}{dx} = 2 + \sqrt{xy}, \text{ given } y = 1, \text{ when } x = 1$$

find y(2) take h = 0.5. (7)

OR

10. (a) Find the root of the equation :

$$\sin x - \frac{x+1}{x-1} = 0, \text{ near to } x = -0.4$$

by Newton-Raphson Method, correct upto three places of decimal. (5)

(b) Solve by Crout's Method :

$$4x + 11y - z = 33$$
$$6x + 3y + 12z = 36$$
$$8x - 3y + 2z = 20$$ (7)

(c) Solve by 4th order Runge-Kutta Method :

$\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$, y =1, when x = 0, h= 0.2 find y(1.2). (6)

11. Formulate and solve by Simplex Method :

A firm can produce three types of cloths say A, B and C. Three kinds of wool are required for it, say red wool, green wool, and blue wool. One unit length of type A cloth needs 2 meters of red wool and 3 meters of blue wool; one unit length of type B cloth needs 3 meters of red wool, 2 meters of green wool and 2 meters of blue wool; and one unit length of type C needs 5 meters of green and 4 meters of blue wool. The firm has only a stock of 8 meters of red wool, 10 meters of green wool and 15. meters of blue wool. It is assumed that the income obtained from one unit length of type A cloth is Rs. 3.00, of type B cloth is Rs. 5.00 and that of type C cloth is Rs. 4.00. Determine how the firm should use the available material so as to maximize the total income from the finished cloth. (12)

OR

12. (a) Use Graphical Method to solve the following L.P. Problem :

Minimize $z = 20x_1 + 10x_2$

subject to $x_1 + 2x_2 \leq 40$

$3x_1 + x_2 \geq 30$

$4x_1 + 3x_2 \geq 60$

$x_1, x_2 \geq 0.$ (5)

(b) Use Big M Method to solve the following L.P. Problem :

Minimize $z = 5x_1 + 3x_2$

subject to $2x_1 + 4x_2 \leq 12$

$2x_1 + 2x_2 = 10$

$5x_1 + 2x_2 \geq 10$

$x_1 , x_2 \geq 0.$ (7)