SUMMER - 2014

APPLIED MATHEMATICS - III

1. Obtain the Fourier series for $f(x) = x - x^2$ in the interval $-1 < x < 1$. (7)

(OR)

2. (a) Obtain half range sine series for $f(x) = \pi x - x^2$ in the interval $(0,\pi)$. (7)

3. (a) Solve $z(p - q) - z^2 + (x + y)^2$. (7)

(b) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = (y - 1)e^x$. (6)

(c) Solve the equation $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given that $u = 3e^{-y} - e^{-5y}$, when $x = 0$ by the method of separation of variables. (7)

(OR)

4. (a) A strecthed string with fixed ends at $x = 0$, $x = \ell$ is initially in a position given by $y(x,0) = a \sin\left(\frac{\pi x}{\ell}\right)$. If it is released from the rest, show that the displacement of any point at a distance x from one end at any time t is given by

$$y(x, t) = a \sin \frac{\pi x}{\ell} \cos \frac{\pi t}{\ell}.$$ (8)

(b) Solve $4\frac{\partial^2 z}{\partial x^2} + 12\frac{\partial^2 z}{\partial x \partial y} + 9\frac{\partial^2 z}{\partial y^2} = e^{3x-2y}$. (5)

(c) Solve $(y - zx)\, p + (x + yz)\, q = x^2 + y^2$. (5)

5. Find the extremals of the functional $v[y(t), z(t)] = \int_0^{\pi/2} [(y')^2 + (z')^2 + 2yz]dt$ given that $y(0) = 0$, $y(/2) = -1$, $z(0) = 0$, $z(/2) = 1$.

(OR)

6. Find the extremals of $I[y(x)] = \int_0^1 (x^2 + t'^2)dx$ given that $\int_0^1 y^2 dx = 2$, $y(0) = 0$, $y(1) = 0$. **(7)**

7. (a) Find whether the following set of vectors are linearly dependent. If dependent, find the relation between them :**(5)**

   $X_1 = (3, 1, -4)$, $X_2 = (2, ,2 -3)$

   $X_3 = (0, -4, 1)$, $X_4 = (-4, -4, 6)$

   (b) Find the model matrix B and verify $B^{-1}AB$ a diagonal matrix if $A = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$. **(6)**

   (c) Verify Cayley-Hamilton theorem and hence find A^{-1}, where

   $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}$. **(7)**

(OR)

8. (a) Using Sylvester's theorm show that $\sin^2 A + \cos^2 A = I$ if

   $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$. **(6)**

   (b) Solve the following differential equation by using matrix method : $\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 0$ given $y(0) = 2$, $y'(0) = 5$ **(6)**

   (c) Find the egien values of the matrix represented by

   $A^3 - 2A^2 + 3a - 2I$. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$. **(6)**

9. (a) Find by Newton-Raphson method the root of the equation $x + \log_{10}x - 3.375 = 0$ corrent uoto three decimal places.**(5)**

   (7)

   (b) Solve by Crout's method :

   $2x - 6y + 8z = 24$

   $5x + 4y - 3z = 2$

   $3x + y + 2z = 16$

(c) Solve by modified Euler's method : $\frac{dy}{dx} = y + e^x$ given $y(0) = 0$ find $y(0.2)$. (6)

(OR)

10. (a) Using Bisection method solve the equation $3x - \cos x = 1$. (5)

(b) Solve by Gauss-Jordan method : (6)

$5x + 2y + z = 12$

$x + 4y + 2z = 15$

$x + 2y + 5z = 10$

(c) By Milne's Predictor-Corrector method $\frac{dy}{dx} = \frac{1}{2}(1 + x^2) y^2$ where $y(0) = 1$, $y(0.1) = 1.06$, $y(0.2) = 1.12$, $y(0.3) = 1.21$ find $y(0.4)$. (7)

11. (a) Solve by graphical method : (6)

Maximize $Z = 4x_1 + 10x_2$

s.t. $2x_1 + x_2 \le 50$

$2x_1 + 5x_2 \le 100$

$2x_1 + 3x_2 \le 90$

$x_1, x_2 \ge 0$

(b) Use big M method to : (6)

Maximize $Z = 6x_1 + 4x_2$

s.t. $2x_1 + 3x_2 \le 30$

$3x_1 + 2x_2 \le 24$

$x_1 + x_2 \ge 3$

$x_1, x_2 \ge 0$

(OR)

12. Formulate andsolve : (12)

A company makes two types of leather belts. Belt A is a high quality belt, and belt B is of lower quality. The respective profits are Rs. 4.00 and Rs. 3.00 per belt. Each belt of type A requires trwice as much time as a belt of type B, and if all belts were of type B, the company could make 1000 belts per day. The supply of leather is sufficient for only 800 belts per day (both A & B combined). Belt A requires a fancy buckle and only 400 buckles per day are available. There are only 700 buckles a day available for belt B. Determine the product mix.

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