Faculty of Engineering & Technology
Third Semester B.E. (Electroni./Electri./ET/EC Mech. Engg.) (C.B.S.) Examination
APPLIED MATHEMATICS—III

Time—Three Hours] [Maximum Marks—80

INSTRUCTIONS TO CANDIDATES

(1) All questions carry marks as indicated.

(2) Solve **SIX** questions as follows :

Que. No. **1** **OR** Que. No. **2**

Que. No. **3** **OR** Que. No. **4**

Que. No. **5** **OR** Que. No. **6**

Que. No. **7** **OR** Que. No. **8**

Que. No. **9** **OR** Que. No. **10**

Que. No. **11** **OR** Que. No. **12**

(3) Use of Non programmable Calculator is permitted.

1. (a) If $L\{f(t)\} = F(s)$, then prove that :

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$$

Hence find $L\{t^2 \sin 3t\}$. 6

(b) Find Inverse Laplace Transform of $\left\{\log\left(1+\frac{1}{s^2}\right)\right\}$.

Hence or otherwise show that :

$$L^{-1}\left\{\frac{1}{s}\log\left(1+\frac{1}{s^2}\right)\right\} = \int_0^t \frac{2}{x}(1-\cos x)\,dx$$

6

OR

2. (a) Express $f(t) = \begin{cases} 1, & 0 \le t < 2 \\ -3, & 2 \le t < 3 \\ t^2, & t \ge 3 \end{cases}$, in terms of unit step function and hence find its Laplace Transform. 6

(b) Solve $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4t + e^{3t}$, given $y(0) = 1$, $y'(0) = -1$, using Laplace Transform technique. 6

3. (a) Obtain Fourier series for $f(x) = \begin{cases} \pi x & ; \ 0 \le x \le 1 \\ \pi(2-x) ; & 1 \le x \le 2 \end{cases}$

Hence show that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$ 6

Contd.

(b) Find Fourier sine transform of $e^{-|x|}$ and hence show that $\int_0^\infty \frac{x \sin mx}{1+x^2}\,dx = \frac{\pi}{2}e^{-m}$, $m > 0$. 6

OR

4. (a) Obtain half range sine series for $f(x) = \pi x - x^2$ in the interval $(0, \pi)$. 6

(b) Using Fourier Integral, show that :

$$\int_0^\infty \frac{\sin \pi\lambda \sin \lambda x}{1-\lambda^2}\,d\lambda = \begin{cases} (\pi/2)\sin x, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}$$

6

5. Find the extremals of the functional

$$\int_1^2 \frac{\sqrt{1+(dy/dx)^2}}{x}\,dx, \text{ given } y(1) = 0,\ y(2) = 1.$$

6

OR

6. Find the extremals of :

$$V(y) = \int_{x_0}^{x_1} \left\{(y'')^2 - 2(y')^2 + y^2 - 2y \sin x\right\} dx$$

6

Contd.

7. (a) If $u = y^3 - 3x^2y$, show that u is harmonic. Find v and the corresponding analytic function $f(z) = u + iv$. 6

(b) Evaluate $\int_C \frac{\cos \pi z^2}{(z-1)(z-2)} dz$ where C is circle $|z+i| = 1.5$, using Cauchy's integral theorem. 6

(c) Expand $f(z) = \frac{z}{(z+1)(z+2)}$ about $z = -2$ in Taylor's series. 6

OR

8. (a) Show that $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta} d\theta = \frac{\pi}{6}$ by contour integration. 7

(b) Evaluate $\int_C \frac{z-1}{(z+1)^2 (z-2)} dz$ $C : |z - i| = 2$ by using Residue theorem. 5

Contd.

(c) Expand $f(z) = (z^2 + 4z + 3)^{-1}$ by Laurentz series valid for :

(i) $1 < |z| < 3$

(ii) $|z| < 1$. 6

9. (a) Solve :

$$x^2(y - z)p + y^2(z - x)q = z^2(x - y)$$ 6

(b) Solve using method of separation of variables, $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = u$, given that $u(x, 0) = 3e^{-5x} + 2e^{-3x}$. 8

OR

10. (a) Solve $(D^2 - 3DD' + 2D'^2)z = e^{2x+3y} + \sin(x - 2y)$. 6

(b) Solve $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, $0 \le x \le \ell, t > 0$.

$u(0, t) = u(\ell, t) = 0$, $u(x, 0) = \frac{100x}{\ell}$ by method of separation of variables. 8

Contd.

11. (a) Investigate the linear dependence and independence of vectors :

$x_1 = (1, 2, 3), x_2 = (3, 1, 2), x_3 = (5, 5, 8)$ 6

(b) Diagonalise the matrix, $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. 6

(c) Solve :

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 10y = 0, \; y(0) = 3, \; y'(0) = 15$$

by matrix method. 6

OR

12. (a) Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find the matrix represented by

$A^7 - 5A^6 + 9A^5 - 13A^4 + 17A^3 - 21A^2 - 81.$ 6

(b) Use Sylvester's theorem to show that :

$3 \tan A = (\tan 3)A$, where $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$. 6

(c) If $\begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} -3 & 5 \\ 7 & 4 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} R \\ S \end{bmatrix}$ and $\begin{bmatrix} R \\ S \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} L \\ B \end{bmatrix}$

express $(J_1 + J_2 - J_3)$ in terms of L and B. 6