Faculty of Engineering & Technology

First Semester B.E. (C.B.S.) Examination

APPLIED MATHEMATICS—I

Paper—I

Time—Three Hours] [Maximum Marks—80

N.B. :— (1) All questions carry marks as indicated.

(2) Use of Non-programmable calculator is permitted.

(3) Solve :

Que. No. 1 **OR** Que. No. 2

Que. No. 3 **OR** Que. No. 4

Que. No. 5 **OR** Que. No. 6

Que. No. 7 **OR** Que. No. 8

Que. No. 9 **OR** Que. No. 10

Que. No. 11 **OR** Que. No. 12

1. (a) If $Y = (\sin^{-1} x)^2$; then prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$. 6

(b) Evaluate $\lim_{x \to 0} \frac{e^x + e^{-x} - 2\cos^x}{x \sin x}$. 3

(c) Evaluate $\lim_{x \to 0}\left[\frac{1}{x} - \cot x\right]$. 3

OR

2. (a) IF $x = a\cos^4\theta$; $y = a\sin^4\theta$; find the curvature at $\theta = \pi/6$. 6

(b) Use Taylor's Series Expansion to evaluate $\sin 60^\circ 30'$ correct up to three decimal places. 6

3. (a) If $v = At^{-1/2} e^{-\frac{x^2}{4a^2t}}$

then prove that

$$\frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial x^2}.$$ 6

(b) If $u = \sin^{-1}\left(\frac{x^2 + y^2}{\sqrt{x} + \sqrt{y}}\right)$

find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}.$$ 6

(c) If $\phi = f(x, y, z)$

and $x = \sqrt{vw}$, $y = \sqrt{wu}$, $z = \sqrt{uv}$

then show that

$$u\frac{\partial \phi}{\partial u} + v\frac{\partial \phi}{\partial v} + w\frac{\partial \phi}{\partial w} = x\frac{\partial \phi}{\partial x} + y\frac{\partial \phi}{\partial y} + z\frac{\partial \phi}{\partial z}.$$ 6

OR

4. (a) If $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_3 x_1}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$

find the value of $\frac{\partial(x_1, x_2, x_3)}{\partial(y_1, y_2, y_3)}$. 6

(b) Expand $e^x \cos y$ in the power of x and y as far as the term of third degree. 6

(c) Given $f(x, y, z) = \frac{5xyz}{x + 2y + 4z}$. Find the values of x, y, z for which f(x, y, z) is maximum. Subject to the condition $xyz = 8$ (by Lagrange method). 6

5. (a) Find the inverse of matrix by partitioning method :

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 3 & 2 \\ 2 & 4 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$ 6

(b) Find the rank of matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}.$$ 6

OR

6. (a) Test the consistency and solve :

$x + y + z = 6$

$x - y + 2z = 5$

$3x + y + z = 8.$ 6

(b) Solve the system of equations by adjoint method :

$2x + 3y + 4z = 15$

$3x - y + 2z = 9$

$x + y + z = 5.$ 6

7. (a) Solve $\frac{dy}{dx} - y \tan x = 3e^{\sin x}$. 4

(b) Solve $3y^2 \frac{dy}{dx} + 2xy^3 = 4xe^{-x^2}$ 4.

(c) Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$. 4

OR

8. (a) Solve $p^2 + 2px + py + 2xy = 0$. 4

(b) Solve $y = x + 2\tan^{-1}p$. 4

(c) A resistance R = 50 ohms and an inductance L = 10 henaries are connected in series with a constant voltage of E = 100 volts. If the current is zero at t = 0. Find :

(i) The equation of i

(ii) The current at t = 0.5 seconds. 4

9. (a) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = e^{2x}\sin 3x$. 6

(b) Solve by method of variation of parameter

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \tan x.$$ 6

(c) Solve $x^3\frac{d^3y}{dx^3} - 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$. 6

OR

10. (a) In an L.C.R. circuit the charge q on a plate of a condenser is given by.

$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=E\sin pt$ The circuit is tuned to resonance so that $P^2=\frac{1}{LC}$. If initially current i and the charge q be zero show that for small value of $\frac{R}{L}$, the current in the circuit at time t is given by $\frac{Et}{2L}\sin pt$. 6

(b) Solve :

$$\frac{dx}{dt}+y-\sin t=0$$

$$\frac{dy}{dt}+x-\cos t=0.$$ 6

(c) Solve $\frac{d^2y}{dx^2}=2(y^3+y)$ under the condition $\dot{y}=0$; $\frac{dy}{dx}=1$ when $x=0$. 6

11. (a) Prove that $\log\tan\left(\frac{\pi}{4}+i\frac{x}{2}\right)=i\tan^{-1}(\sinh x)$. 4

Contd.

(b) Find all the values of $\left[\frac{1}{2}+\frac{\sqrt{3}}{2}i\right]^{3/4}$ and show that the continued product of all the values is 1. 4

OR

12. (a) Solve using De-Moivre's theorem $x^7-x^4+x^3-1=0$. 4

(b) Separate $\tan^{-1}(x+iy)$ into real and imaginary parts. 4