Faculty of Engineering & Technology

First Semester B.E. (CBS) Examination

APPLIED MATHEMATICS-I

Paper-I

Time—Three Hours] [Maximum Marks—80

INSTRUCTIONS TO CANDIDATES

(1) Solve **SIX** questions as follows :

Que. No. - 1 **OR** Que. No. - 2

Que. No. - 3 **OR** Que. No. - 4

Que. No. - 5 **OR** Que. No. - 6

Que. No. - 7 **OR** Que. No. - 8

Que. No. - 9 **OR** Que. No. - 10

Que. No. - 11 **OR** Que. No. - 12

(2) Use of non-programmable calculator is permitted.

1. (a) If $y = a \cos(\log x) + b \sin(\log x)$

show that $x^2y_2 + xy_1 + y = 0$ and

$x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. 6

(b) Evaluate :

$$\lim_{x \to 0} \left(\frac{1}{\sin^2 x} - \frac{1}{x^2} \right)$$ 3

$$\lim_{x \to 0} x \tan\left(\frac{\Pi}{2} - x \right)$$ 3

OR

2. (a) If $x = a\cos^4\theta$, $y = a\sin^4\theta$, find the curvature of $\theta = \frac{\Pi}{6}$ — 7

(b) Expand $3x^3 - 2x^2 + x - 4$ in powers of $(x-2)$. — 5

3. (a) If $u = \log[\tan x + \tan y + \tan z]$.

Prove that

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$$ — 5

(b) If $u = \sin^{-1}\left[\frac{x^2 + y^2}{\sqrt{x} + \sqrt{y}}\right]$, then

find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ — 7

(c) Given $u = \sin^{-1}x + \sin^{-1}y$, and $v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$. Are u, v functionally related ? If so, find the relation between them. — 6

OR

4. (a) If $u = \frac{yz}{x}, v = \frac{xz}{y}, w = \frac{xy}{z}$

Find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ — 6

(b) Expand $x^2y + 2y - x^2 - 2$ in powers of $(x-1)$ and $(y+1)$ by Taylor's theorem. — 6

(c) The temperature T at any point (x, y, z) in space is $T = 400\,xyz^2$. Find the highest temperature on the surface $x^2 + y^2 + z^2 = 1$. — 6

5. (a) Test the following system for consistency and solve it :

$$x + y + z = 6$$
$$2x + y + 3z = 13$$
$$5x + 2y + z = 12$$ — 6

(b) Find the inverse of the following matrix by partitioning method :

$$\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$ — 6

OR

6. (a) Find the rank of the following matrix :

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}$$ — 6

(b) By adjoint method solve the system of equations :

$$x + y + z = 3$$
$$x + 2y + 3z = 4$$
$$x + 4y + 9z = 6$$ 6

7. (a) Solve : $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$. 4

(b) Solve : $\frac{dy}{dx} + xy = x^3y^3$. 4

(c) Solve : $\frac{dy}{dx} = -\frac{xy^2}{2 + x^2y}$. 4

OR

8. (a) Solve : $p^3 - 4xyp + 8y^2 = 0$. 3

(b) Solve : $x^2(y - px) = yp^2$. 3

(c) When a resistance R ohms is connected in series with an inductance L henries, an e.m.f. of E volts and current , amperes of time t is given by :

$$L\frac{di}{dt} + Ri = E.$$

If E = 10 sin t volts and i = 0 when t = 0, find i as a function of t. 6

9. (a) Solve : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \sin 2x$. 6

(b) Solve : $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ by method of variation of parameters. 6

(c) Solve : $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x \log x$. 6

OR

10. (a) Solve the simultaneous differential equation :

$$\frac{dx}{dt} + 3x - 2y = 1$$
$$\frac{dy}{dt} - 2x + 3y = e^t.$$ 6

(b) Solve $\frac{d^2y}{dx^2} = 3\sqrt{y}$, given that $y = 1$, $\frac{dy}{dx} = 2$ when $x = 0$. 6

(c) The differential equation of simple pendulum is $\frac{d^2x}{dt^2} + w_0^2x = F_0 \sin nt$, where W_0 and F_0 are constants. If initially $x = 0$, $\frac{dx}{dt} = 0$, determine the motion when $w_0 \neq n$. 6

11. (a) Find all values of $(1 + i)^{2/3}$. 4

(b) If $2 \cos\theta = x + \frac{1}{x}$, $2 \cos\phi = y + \frac{1}{y}$

prove that $x^my^n + \frac{1}{x^my^n} = 2\cos(m\theta + n\phi)$. 4

OR

12. (a) Use De - Moivre's theorem to solve $x^5 + 1 = 0$. 4

(b) If $\cos(\theta + i\phi) = R(\cos\alpha + i\sin\alpha)$

prove that $\phi = \frac{1}{2}\log\left[\frac{\sin(\theta-\alpha)}{\sin(\theta+\alpha)}\right]$ 4