Faculty of Engineering & Technology

First Semester B.E. (C.B.S.) Examination

APPLIED MATHEMATICS—I

Paper—I

Time—Three Hours] [Maximum Marks—80

INSTRUCTIONS TO CANDIDATES

(1) All questions carry marks as indicated.

(2) Use of Non-programmable calculator is permitted.

(3) Solve Q.No. 1 OR Q.No. 2

Q.No. 3 OR Q.No. 4

Q.No. 5 OR Q.No. 6

Q.No. 7 OR Q.No. 8

Q.No. 9 OR Q.No. 10

Q.No. 11 OR Q.No. 12.

1. (a) If $y = \sin \log (x^2 + 2x + 1)$, prove that :

(i) $(x + 1)^2 y_2 + (x + 1)y_1 + 4y = 0$

(ii) $(x+1)^2 y_{n+2} + (2n+1)(x+1) y_{n+1} + (n^2+4) y_n = 0$ 6

(b) Evaluate $\lim_{x \to 0} \dfrac{\sin x^2 - \sin^2 x}{x^4}$. 3

(c) Evaluate $\lim_{x \to 0} (\cot x)^{\sin x}$. 3

OR

2 (a) Find radius of curvature, centre of curvature and equation to the circle of curvature for the curve $xy(y-x) = 2$ at the point $(1, -1)$. 7

(b) Using Taylor's series, find the value of cos 64° correct upto four decimal places. 5

3. (a) If $\theta = t^n e^{-r^2/4t}$, find what value of n will make

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}.$$ 6

(b) If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, then find the value of

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}.$$ 6

(c) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, find the value of

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}.$$ 6

OR

4. (a) Show that the functions :

$$u = \sin^{-1}x + \sin^{-1}y$$

and $v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$

are functionally related. Find the relation between them. 6

(b) Expand $e^x \log(1+y)$ in the neighbourhood of origin by Taylor's series upto six terms of the expansion. 6

(c) Find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$ 6

5. (a) Find the inverse of matrix by partitioning method :

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}.$$ 6

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(b) Find the rank of matrix :

$$A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}.$$ 6

OR

6. (a) Investigate the values of λ & μ so that the system of equations :

$2x + 3y + 5z = 9$

$7x + 3y - 2z = 8$

$2x + 3y - \lambda z = \mu$ have

(i) No solution

(ii) Unique solution

(iii) Infinite solutions. 6

(b) Solve the system of equations :

$x + y + z = 3$

$x + 2y + 3z = 4$

$x + 4y + 9z = 6$

by adjoint method. 6

7. (a) Solve $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)e^x \sec y$. 4

(b) Solve $(1 + y^2)\, dx = (\tan^{-1}y - x)\, dy$. 4

(c) Solve :

$$(y^2e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0.$$ 4

OR

8. (a) Solve $xy^2 (p^2 + 2) = 2py^3 + x^3$. 3

(b) Solve $y = 2px + p^4x^2$. 3

(c) A 20 ohm resistor is connected in series with a capacitor of 0.01 farad and emf E volt given by $40e^{-3t} + 20e^{-4t}$. If $q = 0$ at $t = 0$, show that the maximum charge on the capacitor is 0.25 coulomb. 6

9. (a) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x.\cos 2x$. 6

(b) Solve $(D^2 + 2D + 1)\, y = 4e^{-x} \log x$ by method of variation of parameters. 6

(c) $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\log x$. 6

OR

10. (a) An emf E sin pt is applied at t = 0 to a circuit containing a capacitance C & inductance L. The current i satisfies the equation

$$L\frac{di}{dt}+\frac{1}{C}\int i\,dt = E\sin pt.$$

If $P^2=\frac{1}{LC}$ and initially the current i and the charge q be zero, show that the current at time t is $\frac{Et}{2L}$ sin pt, where $i=\frac{dq}{dt}$. 7

(b) A mechanical system with two degree of freedom satisfies the equations :

$$2\frac{d^2x}{dt^2}+3\frac{dy}{dt}=4,\; 2\frac{d^2y}{dt^2}-3\frac{dx}{dt}=0.$$

Obtain expressions for x and y in terms of t, given x, y, $\frac{dx}{dt},\frac{dy}{dt}$ all vanish at t = 0. 6

(c) Solve $\frac{d^2y}{dx^2}=\sec^2 y\tan y$, given that y = 0 and $\frac{dy}{dx}=1$, when x = 0. 5

11. (a) Using De-Moivre's theorem, solve :

$$x^5+x^4+x^3+x^2+x+1=0.$$ 4

(b) If sin (A + iB) = x + iy, then prove that :

(i) $\frac{x^2}{\cosh^2 B}+\frac{y^2}{\sinh^2 B}=1$

(ii) $\frac{x^2}{\sin^2 A}-\frac{y^2}{\cos^2 A}=1$. 4

OR

12. (a) Prove that :

$$\cos^6\theta+\sin^6\theta=\frac{1}{8}(3\cos 4\theta+5).$$ 4

(b) Find the modulus and argument of $(1+i)^{1-i}$. 4