Faculty of Engineering & Technology

Third Semester B.E. (Computer Technology)/C.S.E. (C.B.S.) Examination

APPLIED MATHEMATICS—III

Time : Three Hours] [Maximum Marks : 80

INSTRUCTIONS TO CANDIDATES

(1) All questions are compulsory.

(2) Assume suitable data wherever necessary.

(3) Solve **SIX** questions as follows :

Question No. **1 OR** Question No. **2**

Question No. **3 OR** Question No. **4**

Question No. **5 OR** Question No. **6**

Question No. **7 OR** Question No. **8**

Question No. **9 OR** Question No. **10**

Question No. **11 OR** Question No. **12**.

(4) Use of non-programmable Calculator is permitted.

1. (a) If $L[f(t)] = F(s)$, then show that $L\left[\frac{1}{t} f(t)\right] = \int_s^{\infty} F(s)\, ds$. Hence find $L\left[\frac{1-\cos t}{t}\right]$. 6

(b) Find $L^{-1}\left[\frac{s^2}{(s^2+a^2)^2}\right]$. 6

OR

2. (a) Find the Laplace transform of the function f(t) given by

$$f(t) = \begin{cases} \sin wt & 0 < t < \pi / w \\ 0 & \frac{\pi}{w} < t < 2\pi / w \end{cases}$$

where $f\left(t + \frac{2\pi}{w}\right) = f(t)$. 6

(b) A particle moves in a line so that its displacement x from a fixed point O at any time t, is given by :

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 5x = 80 \sin 5t.$$

Using Laplace transform, find its displacement at any time t if x and x′ vanish at t = 0. 6

3. (a) Find a Fourier series to represent $(x - x^2)$ from $x = -\pi$ to π and hence show that :

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots \qquad 6$$

(b) Find Fourier transform of $e^{-a|x|}$, where $a > 0$. 6

OR

4. (a) Obtain half range cosine series for $f(x)$, $f(x) = 2x - 1;\ 0 < x < 1$. Hence show that :

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots \qquad 6$$

(b) Find Fourier transform of $f(x)$, where

$$f(x) = \begin{cases} 1 - |x| & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases} \text{ and}$$

hence find the value of $\int_0^\infty \frac{\sin t}{t} dt$. 6

5. (a) If $Z[f(n)] = F(z)$, then show that $Z[n.f(n)] = -z\frac{d}{dz}F(z)$. Hence show that $Z\{n^2\} = \frac{z(z+1)}{(z-1)^3}$.

6

(b) Solve by using Z-transform :

$$y_{n+2} + 5y_{n+1} + 6y_n = 6^n,\ y_0 = 0,\ y_1 = 1.$$ 6

OR

6. (a) Find $Z^{-1}\left[\dfrac{a\,z(z+a)}{(z-a)^3}\right]$. 6

(b) Find $Z[9^n \cdot \cos n\,\theta]$. 6

7. (a) Prove that $u = e^{-x}(x \sin y - y \cos y)$ is a harmonic function. Hence construct analytic function f(z). 7

(b) Evaluate $\oint\limits_C \dfrac{z+4}{z^2+2z+5}\,dz$, where C is the circle $|z+1| = 1$. 7

OR

8. (a) Expand the function $f(z) = \dfrac{z^2-1}{(z+2)(z+3)}$ in the regions (i) $|z| < 2$ (ii) $2 < |z| < 3$ by Laurent's series. 6

(b) Evaluate $\displaystyle\int_0^{\pi} \frac{1}{3+2\cos\theta}\,d\theta$, by contour integration. 8

9. (a) State and prove Cayley-Hamilton theorem for matrix A. Hence find A^{-1}, where :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 4 & 12 \end{bmatrix}$$ 6

(b) Are the following vectors linearly dependent ? If so, find the relationship between them :

$X_1 = [1, 2, 4]$, $X_2 = [2, -1, 3]$, $X_3 = [0, 1, 2]$, $X_4 = [-3, 7, 2]$. 6

(c) Use Sylvester's theorem to show that $\sin^2 A + \cos^2 A = I$, where $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$. 6

OR

10. (a) Diagonalise the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$. 6

(b) Find largest eigen value by iteration method of the matrix :

$$A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$ 6

(c) Solve $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 10y = 0$ given $y(0) = 3$, $y'(0) = 15$ by matrix method. 6

11. (a) A random variable X has the following probability distribution :

X	f(x)
0	a
1	3a
2	5a
3	7a
4	9a
5	11a
6	13a
7	15a
8	17a

(i) Determine the value of a

(ii) $P(x \leq 4)$

(iii) $P(x > 5)$

(iv) Distribution function. 6

(b) In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total. Of their output 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective, what is the probability that it was manufactured by machine B ? 6

OR

12\. (a) Find moment generating function and first four moment about origin of random variable x, whose density function is given by :

$$f(x) = \begin{cases} \dfrac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases} \qquad 6$$

(b) The mean grade on a final examination was 72 and the standard deviation was 9. The top 10% of the students are to receive 'A' grade. What is the minimum grade a student must get in order to receive an A ? 6