

Faculty of Engineering & Technology

Third Semester B.E. (Electronic./Electric. ET/EC/EN/
Mech. Engg.) (C.B.S.) Examination

APPLIED MATHEMATICS-III

Time—Three Hours]

[Maximum Marks—80

INSTRUCTIONS TO CANDIDATES

- (1) Solve SIX questions as follows : Q.1 or Q.2,
Q.3 or Q.4, Q.5 or Q.6, Q.7 or Q.8, Q.9 or Q.10,
Q.11 or Q.12
- (2) Use of non-programmable calculator is permitted.

1. (a) Using L.T. method, find $\alpha > 0$ if

$$\int_0^{\infty} e^{-2t} [\sin(\alpha+t) + \sin(\alpha-t)] dt = \frac{4}{5} \quad 6$$

- (b) Find :

$$L^{-1} \left[\int_1^{\infty} \cot^{-1} \left(\frac{u}{2} \right) du \right] \quad 6$$

OR

2. (a) Sketch and discuss the waveform of $f(t)$ if :

$$f(t) = L^{-1} \left[\frac{1}{S(1-e^{-5})} \right] \quad 7$$

- (b) Solve the simultaneous differential equations by Laplace Transform method :

$$\frac{dy}{dt} + ay = x$$

$$\frac{dx}{dt} + ax = y$$

3. subject to $x(0) = 0$ and $y(0) = 1$. 5

(a) Find the Fourier series for the function :

$$f(x) = 1 + \frac{2x}{\pi}, -\pi < x \leq 0$$

$$= 1 - \frac{2x}{\pi}, 0 \leq x < \pi$$

Hence show that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$
 6

(b) Obtain half range Fourier sine series for the function :

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 < x < 2 \end{cases}$$
 6

OR

4. (a) Find the Fourier transform of

$$f(x) = \begin{cases} 1-x^2 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$$

and hence find

$$\int_0^{\pi} \left(\frac{\sin x - x \cos x}{x^3} \right) \cos \left(\frac{x}{2} \right) dx$$
 6

(b) Solve the integral equation

$$\int_0^t f(x) \sin xt \, dx = \begin{cases} 1, & 0 \leq t < 1 \\ 2, & 1 \leq t < 2 \\ 0, & t > 2 \end{cases}$$

5. Determine a function $y(x)$ such that the functional

$$I = \int_0^1 [x^2 + (y')^2] \, dx \text{ is extremum given } \int_0^1 y^2 \, dx = 2$$
$$y(0) = 0 = y(1). \quad 6$$

OR

6. Find the extremals of the function $\int_1^2 \sqrt{1 + (dy/dx)^2} \, dx$,
given $y(1) = 0, y(2) = 1$. $\sqrt{2}, \sqrt{3}$ ① 6

7. (a) If $f(z)$ is analytic function of z then prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2. \quad 6$$

(b) Evaluate

$$\oint_C \frac{\cos \pi z^2}{(z-1)(z-2)} \, dz$$

where C is a circle

(i) $|z| = 3$

(ii) $|z + i| = 1.5$ 6

(c) Find the Laurent's series expansion of the function

$$f(z) = \frac{z^2 - 1}{(z+2)(z+3)} \text{ in the region } (1-2h) \dots \frac{1}{3} \dots 1-2h$$

(i) $|z| < 2$ $1 + \frac{3}{2} (1-2h)$

(ii) $2 < |z| < 3$ $1 + \frac{3}{2} (1-2h)$

(iii) $|z| > 3$ $1 + \frac{3}{2} (1-2h)$

OR

8. (a) Evaluate by contour integration $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4 \cos \theta} d\theta$. 6

(b) Expand in Taylor's series $f(z) = \frac{z}{(z+1)(z+2)}$ about $z = 2$. Also find the region of convergence. 6

(c) Show that $u = e^{-x} (x \sin y - y \cos y)$ is harmonic. Find v such that $f(z) = u + iv$ is analytic function. 6

9. (a) Solve :

$$(x - y) p + (x + y) q = 2xz$$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. 7

(b) Solve :

$$\frac{\partial^4 z}{\partial x^4} + 2 \frac{\partial^4 z}{\partial x^2 \partial y^2} + \frac{\partial^4 z}{\partial y^4} = 5x^2 + 8y^2 + 3xy. \quad 7$$

OR

10. (a) Solve the equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$$

given that $u = 0$, $\frac{\partial u}{\partial x} = 1 + e^{-3y}$ when $x = 0$ for all values of y , using method of separation of variable. 7

(b) Solve using Laplace transform method :

$$\frac{\partial u}{\partial t} + x \frac{\partial u}{\partial x} = x, x > 0, t > 0$$

$$u(x, 0) = 0, u(0, t) = 0$$

7

11. (a) If

$$A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(6)

prove that

$$A^4 - 2A^3 - 9A - 5I = 7A^2 - 5A - 5I.$$

6

(b) Investigate the linear dependence of the vector

$$X_1 = (1, 2, 4), X_2 = (2, -1, 3)$$

$$X_3 = (0, 1, 2), X_4 = (-3, 7, 2)$$

and if possible, find the relation between them. 6

(6)

1, 5, 5

(c) Find the eigenvalues, eigenvectors and modal matrix of

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & -1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

6

OR

12. (a) Solve the equation using matrix method :

$$\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0$$

$$\text{given } y(0) = 2, y'(0) = 5.$$

6

(b) Use Sylvester's theorem to show that

$$e^{Ax} = e^x \begin{bmatrix} \cosh x & \sinh x \\ \sinh x & \cosh x \end{bmatrix},$$

$$\text{where } A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$$

6

(c) Diagonalise the matrix A by orthogonal transformation :

$$A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$

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