B.E. (Electronics Engineering / Electrical Engineering (Electronics & Power) / Electronics Telecommunication Engineering / Electronics Communication Engineering / Mechanical Engineering) Third Semester (C.B.S.)

Applied Mathematics - III Paper - I

P. Pages : 2 **TKN/KS/16/7300/7305/7310/7315**

Time : Three Hours Max. Marks : 80

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Notes :
1. All questions carry marks as indicated.
2. Attempt six questions as follows.
   Question 1 OR Questions No. 2.
   Question 3 OR Questions No. 4.
   Question 5 OR Questions No. 6.
   Question 7 OR Questions No. 8.
   Question 9 OR Questions No. 10.
   Question 11 OR Questions No. 12.
3. Assume suitable data whenever necessary.
4. Use of non programmable calculator is permitted.

1. a) If $L\{f(t)\} = F(s)$ then prove that $L\left\{\frac{f(t)}{t}\right\} = \int_S^\infty F(s)\,ds$ and hence find $L\left\{\frac{\sin h\,t}{t}\right\}$. **6**

b) Find $L^{-1}\left\{\frac{1}{S(S^2+1)}\right\}$ by convolution Theorem. **6**

OR

2. a) Express $f(t) = \begin{cases} \cos t & , 0 < t < \pi \\ \cos 2t & , \pi < t < 2\pi \\ \cos 3t & , t > 2\pi \end{cases}$ **6**

interms of unit step function and find its Laplace transform.

b) Solve $[D^2 + 2D + 5]\, y = e^{-t} \sin t$, given $y(0) = 0,\ y'(0) = 1$ where $D = \frac{d}{dt}$. **6**

3. a) Find the half range cosine series for sin x when $o < x < \pi$, hence deduce that **6**

$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ---- = \frac{\pi}{4}$.

b) Find Fourier transform of $f(x) = \begin{cases} 1 & , |x| < 1 \\ 0 & , |x| > 1 \end{cases}$ and hence evaluate $\int_O^\infty \frac{\sin x}{x}\,dx$. **6**

OR

4. a) Using the Fourier sine integral show that $\int_0^\infty \frac{w \sin(xw)}{1+w^2}\,dw = \frac{\pi}{2} e^{-x}$, $x > 0$. **6**

b) Find the Fourier series to represent $f(x) = x^2 - 2$, $-2 \le x \le 2$. **6**

5. Prove that shortest distance between two points is a straight line. **6**

OR

6. Find the extremals of $v[y(x)] = \int_{x_0}^{x_1} \left\{x^2 (y')^2 + 2y^2 + 2xy\right\} dx$. **6**

7. a) If $u = y^3 - 3x^2 y$, show that u is harmonic find v and the corresponding analytic function $f(z) = u + iv$. **6**

b) Evaluate $\oint_C \frac{z+4}{z^2 + 2z + 5} dz$ where C is a circle $|z+1| = 1$. **6**

c) Expand $f(z) = \frac{z}{(z+1)(z+2)}$ about z = 2. Also find the region of convergence. **6**

OR

8. a) If $u + v = e^x [\cos y + \sin y]$ find analytic function $f(z) = u + iv$. **6**

b) Using Contour integration evaluate $\int_0^{2\pi} \frac{1}{1 - 2a \sin\theta + a^2} d\theta, \; 0 < a < 1$ **6**

c) By using Cauchy Residue Theorem evaluate $\int_C \frac{\sin^6 z}{\left(z - \frac{\pi}{6}\right)^3} dz$ where C is a circle $|z| = 1$. **6**

9. a) Solve : $xq = yp + xe^{(x^2+y^2)}$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ **6**

b) Solve : $\left[D^3 - 3DD'^2 - 2D'^3\right] z = \cos(x + 2y) - e^y (3 + 2x)$ where $D = \frac{\partial}{\partial x}, D' = \frac{\partial}{\partial y}$ **8**

OR

10. a) Solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$ given that $u = 0, \frac{\partial u}{\partial x} = 1 + e^{-3y}$ when $x = 0$ using method of separation of variables. **7**

b) Solve by using Laplace transform. **7**

$\frac{\partial U}{\partial t} + x \frac{\partial U}{\partial x} = x$, $x > 0, t > 0$, $U(x,0) = 0$ and $U(0, t) = 0$

11. a) Investigate the linear dependence of vectors $x_1 = (2, -1, 3, 2)$, $x_2 = (1, 3, 4, 2)$, $x_3 = (3, -5, 2, 2)$ and if possible find the relation between them. **6**

b) Find the Modal Matrix for the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. **6**

c) If $A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$. verify $\log_e e^A = A$ by Sylvester's Theorem. **6**

OR

12. a) Using Cayley Hamilton's Theorem find A^8 if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$. **6**

b) Solve $\frac{d^2x}{dt^2} + 4x = 0$ given $x(0) = 1, x'(0) = 0$ by matrix method. **6**

c) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy + 4zx - 2yz$ to the canonical form by an orthogonal transformation. **6**

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