B.E.(Aeronautical Engineering) Semester Third (C.B.S.)

Applied Mathematics - III

Paper - I

P. Pages : 3 **KNT/KW/16/7258**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data wherever necessary.
9. Illustrate your answers wherever necessary with the help of neat sketches.
10. Use of non programmable calculator is permitted.

1. a) If $L\{f(t)\} = \overline{f}(s)$ **6**

then prove that $L\{t^n f(t)\} = (-1)^n \dfrac{d^n}{ds^n}\overline{f}(s)$

and hence find $L\{t \sin t\}$

b) Find $L^{-1}\left\{\dfrac{1}{(s+1)(s^2+1)}\right\}$ by convolution theorem. **6**

OR

2. a) Express : **6**

$$f(t) = \begin{cases} 1 & , \ 0 \le t < 2 \\ -3 & , \ 2 \le t < 3 \\ t^2 & , \ t \ge 3 \end{cases}$$

in terms of unit step function and hence find its Laplace transform.

b) Solve the equation by using Laplace transform **6**

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t,$$

given $y(0) = 0,\ y'(0) = 1.$

3. a) Find Fourier transform of **6**

$$f(x) = \begin{cases} 1 & , \ \text{for } |x| < 1 \\ 0 & , \ \text{for } |x| > 1 \end{cases}$$

hence find $\displaystyle\int_0^\infty \frac{\sin x}{x}\,dx$

OR

4. a) Solve the integral equation **6**

$$\int_0^{\infty} f(x)\cos(\alpha x)\,dx = \begin{cases} 1-\alpha\ , & 0 \le \alpha \le 1 \\ 0, & \alpha > 1 \end{cases}$$

and hence evaluate $\int_0^{\infty} \frac{\sin^2 t}{t^2}\,dt$.

5. a) If $u = y^3 - 3x^2 y$, show that u is harmonic. Find V and the corresponding analytic function $f(z) = u + iv$ in terms of z. **7**

b) Evaluate $\oint_C \frac{\cos \pi z^2}{(z-1)(z-2)}\,dz$, **7**

where C is a circle (i) $|z| = 3$

(ii) $|z+i| = 1.5$

OR

6. a) Find the Laurent's series expansion of the function $f(z) = \frac{1}{(z-1)(z-2)}$ in the region **7**

(i) $1 < |z| < 2$ (ii) $0 < |z-1| < 1$ (iii) $|z| > 2$.

b) Evaluate : $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta}\,d\theta$ **7**

by contour integration.

7. a) Solve : $\left(x^2 - y^2 - z^2\right)p + 2xyq = 2xz$ **6**

where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$

b) Solve : $\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x \partial y^2} - 6\frac{\partial^3 z}{\partial y^3} = \sin(x+2y)$ **6**

OR

8. a) Solve the equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, **6**

given that $u(x,0) = 6e^{-3x}$ by method of separation of variables.

b) A rod of length ℓ with insulated sides is initially at uniform temperature u_0, its ends are suddenly cooled at 0ºC and kept at that temperature. Find the temperature function $u(x,t)$, if it satisfies the equation $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$. **6**

9. a) Are the following vectors Linearly dependent? if so, find the relation between them **6**

$$X_1 = [1,1,1,3], \; X_2 = [1,2,3,4] \; X_3 = [2,3,4,7]$$

b) Diagonalise the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. **6**

c) Verify Caley - Hamilton theorem and express $A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$ as a Linear polynomial in A, if $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$. **6**

OR

10. a) Using Sylvester's theorem find, A^{-1} where $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$. **6**

b) Solve the Differential equation by matrix method $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 3y = 0$, given $y(0) = 2$, $y'(0) = 2$. **6**

c) Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ to canonical form by orthogonal transformation. **6**

11. a) Using Regula Falsi method, find the root of equation $3x - \cos x = 1$ correct to three decimal places. **6**

b) Solve the system of equation by Gauss Seidal method. **6**

$x + 7y - 3z = -22$, $\quad 5x - 2y + 3z = 18$,
$2x - y + 6z = 22$.

c) Using Euler's modified method, solve the equation $\frac{dy}{dx} = -xy^2$, $y = 2$ when x = 0, find $y(0.2)$ taking h = 0.1 **6**

OR

12. a) Find the real root of $x \log_{10} x - 2 = 0$ by Newton - Raphson method, correct upto three places of decimal. **6**

b) Solve the following system of equation by Cront's method **6**

$4x + y - z = 13$, $3x + 5y + 2z = 21$,
$2x + y + 6z = 14$.

c) Solve by Runge - Kutta fourth order method $\frac{dy}{dx} = \frac{y - x}{y + x}$, $y(0) = 1$, find $y(0.2)$ by taking $h = 0.2$. **6**

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