B.E. Second Semester All Branches (C.B.S.) / B.E. Second Semester (Fire Engineering)

Applied Mathematics - II

P. Pages : 3 **NKT/KS/17/7202**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.

1. a) Prove that $\int_0^1 x^{n-1}\left(\log\frac{1}{x}\right)^{m-1} dx = \frac{\overline{|m}}{n^m}$ **6**

b) Evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x} dx,\ \alpha \ge 0$ by differentiating under integral sign. **6**

OR

2. a) Evaluate $\int_0^1 x^4 (1-\sqrt{x})^5 dx$ **6**

b) Find Root Mean square value of $\log_e{}^x$ over the range x = 1 to x = e. **6**

3. a) Trace the curve $y^2(2a-x)=x^3$. **6**

b) Find the area enclosed by two parabolas $y^2 = 4x$ and $y^2 = -4(x-2)$. **6**

OR

4. a) Trace the curve $r = a(1+\cos\theta)$ and find the perimeter of the curve. **6**

b) Find the volume of the solid obtained by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about x axis. **6**

5. a) Evaluate $\iint_R y\,dx\,dy$ where R is the region bounded by parabolas $y^2 = 4x$ and $x^2 = 4y$. **6**

b) Evaluate by changing order of integration. **6**

$$\int_0^4\int_y^4 \frac{x}{x^2+y^2}\,dy\,dx$$

c) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{\sqrt{x^2+y^2}}\,dy\,dx$ by changing into polar coordinates. **6**

OR

6. a) Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z} (x+y+z)\,dy\,dx\,dz$ **6**

b) Evaluate $\iint r^3 dr d\theta$ over the area bounded by circles $r = 2\cos\theta$ and $r = 4\cos\theta$. **6**

c) Find the area lying between the parabola $y = 4x - x^2$ and the line y = x. **6**

7. a) Prove that **6**

i) $[\bar{b}-\bar{c} \;\; \bar{c}-\bar{a} \;\; \bar{a}-\bar{b}] = 0$

ii) $(\bar{b}\times\bar{c}) \cdot \{(\bar{c}\times\bar{a}) \times (\bar{a}\times\bar{b})\} = \{(\bar{a}\times\bar{b})\cdot c\}^2$

b) A particle moves along a curve $x = t^3+1$, $y = t^2$, $z = 2t+5$, where t is the time. Find the component of its velocity and acceleration at t = 1 in the direction i + j + 3k. **6**

c) Find the angle between the tangents to the curve $\bar{r} = t^2 i - 2tj + t^3 k$ at the points t = 1 and t = 2. **6**

OR

8. a) Find the directional derivative of $\phi = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of line PQ where Q is the point (5, 0, 4). In what direction will it be maximum. **6**

b) A vector field is given by - **6**

$\bar{A} = (x^2 + xy^2)i + (y^2 + x^2 y)j$

Show that field is irrotational and find its scalar potential.

c) If $\bar{r} = xi + yj + zk$ show that **6**

i) $\text{grad}\, r = \frac{\vec{r}}{r}$

ii) $\nabla r^n = n r^{n-2} \vec{r}$

9. Find the total work done in moving a particle in a field of force given by **7**

$\bar{F} = 3xyi - 5zj + 10xk$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from t = 1 to t = 2.

OR

10. Verify Greens' theorem in the plane for $\int_C (3x^2 - 8y^2)\,dx + (4y - 6xy)dy$ where C is the boundary of the region defined by $y = \sqrt{x},\ y = x^2$. **7**

11. a) Fit a curve $y = a + bx^2$ for the following data : **7**

x	0	1	2	3
y	2	4	10	15

b) Using Lagrange's interpolation formula, find the value of y when x = 10 from the following table. **6**

x	5	6	9	11
y	12	13	14	16

OR

12. a) The two lines of regressions are **7**

$8x - 10y + 66 = 0\ :\ 40x - 18y = 214$

If $\sigma_x{}^2 = 9$ Find :

i) Mean values of x and y

ii) Coefficient of correlation, and

iii) σ_y, the standard deviation of y.

b) Solve $u_{n+2} - 2u_{n+1} + u_n = n^2 \cdot 2^n$. **6**

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