

P. Pages : 4

Time : Three Hours

**NKT/KS/17/7278**

Max. Marks : 80

- Notes :
1. All questions carry marks as indicated.
 2. Solve Question 1 OR Questions No. 2.
 3. Solve Question 3 OR Questions No. 4.
 4. Solve Question 5 OR Questions No. 6.
 5. Solve Question 7 OR Questions No. 8.
 6. Solve Question 9 OR Questions No. 10.
 7. Solve Question 11 OR Questions No. 12.
 8. Assume suitable data whenever necessary.
 9. Illustrate your answers whenever necessary with the help of neat sketches.
 10. Use of non programmable calculator is permitted.
 11. Table for the area under standard normal curve is permitted.

1. a) Define : 6
- i) Step signal
 - ii) Ramp signal and
 - iii) Parabolic signal.
- Also find their Laplace transforms.

- b) A unity feedback system is characterized by an open loop transfer function 6
- $$G(s) = \frac{k}{s(s+10)}$$

Determine the gain k so that the system will have a damping ratio of 0.5. For this value of k , determine the rise time, peak time and settling time for a unit step input.

OR

2. a) Define the transfer function obtain the mathematical model and transfer function for a series R-C circuit. 6

- b) Obtain unit step response of unity feedback system whose open loop transfer function is - 6
- $$G(s) = \frac{4}{s(s+5)}$$

3. a) If $Z\{f(n)\} = F(z)$, prove that $Z\{f(n+k)\} = z^k \left[F(z) - \sum_{m=0}^{k-1} f(m)z^{-m} \right]$, where $k > 0$. 6

Hence find $Z\left\{\frac{1}{(n+1)!}\right\}$

- b) Prove that $Z^{-1}\{e^{2/z}\} = \frac{2^n}{n!}$ by convolution theorem. 6

OR

4. a) Find inverse Z - transform of $\frac{z^4}{(z-a)^4}$ using Residue method. 6

b) Solve $x_{n+2} + 3x_{n+1} + 2x_n = u_n$, given that $x_0 = 1$ and $x_n = 0$ for $n < 0$. 6

5. a) Define : 6

- i) Fuzzy set
- ii) Support of fuzzy set,
- iii) α -cut and
- iv) normalized fuzzy set

b) If the universe of discourse is $X = \{0, 1, 2, 3, 4, 5\}$, then write the fuzzy set A whose membership grade function is 6

$$\mu_A(x) = \frac{x}{x+2}$$

Also find \bar{A} , scalar cardinality of A and 0.2 cut of A.

OR

6. a) Let $\cup = \{a, b, c, d\}$ be the domain and A and B be two fuzzy sets on \cup as given below : 6

	a	b	c	d
A	0.5	0.8	0.0	0.3
B	0.2	1.0	0.1	0.7

Find

i) $A \cup B$ ii) $A \cap B$, iii) A' and iv) B'

b) Let $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3, y_4, y_5\}$ and $Z = \{z_1, z_2, z_3, z_4\}$. The fuzzy relations R and S are defined as follows : 6

R	y_1	y_2	y_3	y_4	y_5
x_1	0.1	0.2	0	0.7	0
x_2	0.3	0.5	0	0.2	1
x_3	0.8	0	1	0.4	0.3

and

S	z_1	z_2	z_3	z_4
y_1	0.9	0	0.3	0.4
y_2	0.2	1	0.8	0
y_3	0.8	0	0.7	1
y_4	0.4	0.2	0.3	0
y_5	0	1	0	0.8

Find ROS.

7. a) Find the root of the equation $xe^x - \cos x = 0$ using Regula-Falsi method. 6

- b) Solve the system of equations using Gauss-Seidal method :
 $4x + 11y - z = 33$,
 $6x + 3y + 12z = 36$
and $8x - 3y + 2z = 20$

6

OR

8. a) Write Newton-Raphson procedure for finding $\sqrt[3]{N}$, where N is a real number. Use it to find $\sqrt[3]{18}$ by assuming 2.5 as initial approximation.

6

- b) Solve the system of following equations using Crouts method :
 $x + y + z = 1$, $3x + y - 3z = 5$ and $x - 2y - 5z = 10$.

6

9. a) Using Euler's modified method, solve the equation

7

$$\frac{dy}{dx} = x + |\sqrt{y}|, \text{ given } y = 1, \text{ when } x = 0 \text{ for the range } 0 \leq x \leq 0.6 \text{ by taking } h = 0.2.$$

- b) Solve the following differential equation by Milne's predictor corrector method :

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$$\frac{dy}{dx} = 1 + xy^2, y(0) = 1, y(0.1) = 1.105, y(0.2) = 1.223, y(0.3) = 1.355 \text{ find } y(0.4) \text{ and } y(0.5).$$

OR

10. a) Solve

6

$$\frac{dy}{dx} = 2y + 3e^x, y(0) = 0 \text{ using Taylor's series method and find } y(0.1) \text{ and } y(0.2)$$

- b) Solve following simultaneous differential equations using Runge-Kutta method :

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$$\frac{dy}{dx} = yz + x, \frac{dz}{dx} = xz + y, \text{ for } x = 0.2, \text{ given } y(0) = 1, z(0) = -1 \text{ and } h = 0.2.$$

11. a) An insurance company insured 2,000 scooter drivers, 4,000 car drivers and 6,000 truck drivers. The respective probabilities of an accident are 0.01, 0.03 and 0.15. One of the ensured persons meets an accident. What is the probability that he is a scooter driver ?

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- b) Find moment generating function and first four moments about origin for the r.v. X having density function

6

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & , x > 0 \\ 0 & , x \leq 0 \end{cases}$$

- c) If 3% of the electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs -

6

- i) at the most 2,
ii) at least 2 and
iii) between 1 and 3 bulbs will be defective

OR

12. a) A random variable X takes the values $-3, -2, -1, 0, 1, 2, 3$ such that
 $P(X = 0) = P(X > 0) = P(X < 0)$,
 $P(X = -3) = P(X = -2) = P(X = -1)$ and
 $P(X = 1) = P(X = 2) = P(X = 3)$.
Find probability function and distribution function.

6

- b) Find coefficient of skewness and Kurtosis for the random variable X given by -

6

$$X = \begin{cases} 1 & , \text{ Pr ob. } \frac{1}{2} \\ -1 & , \text{ Pr ob. } \frac{1}{2} \end{cases}$$

- c) Find the probability of getting between 2 heads to 4 heads in 10 tosses of fair coin using -
i) Binomial distribution and
ii) Normal approximation to the Binomial distribution.

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