

Applied Mathematics - IV

P. Pages : 3

Time : Three Hours



NKT/KS/17/7268/7273

Max. Marks : 80

- Notes :
1. All questions carry marks as indicated.
 2. Solve Question 1 OR Questions No. 2.
 3. Solve Question 3 OR Questions No. 4.
 4. Solve Question 5 OR Questions No. 6.
 5. Solve Question 7 OR Questions No. 8.
 6. Solve Question 9 OR Questions No. 10.
 7. Solve Question 11 OR Questions No. 12.
 8. Use of non programmable calculator is permitted.
 9. Use of normal distributed table is permitted.

1. a) Use Newton-Raphson method to find the real root of the equation $3x = \cos x + 1$ correct to four decimal places. **6**
- b) Solve the following system of equation by Crout's method. **6**
- $$\begin{aligned} 2x + 3y + z &= 9 \\ x + 2y + 3z &= 6 \\ 3x + y + 2z &= 8 \end{aligned}$$
- c) Use Taylor's series method to solve **6**
- $$\frac{dy}{dx} = x + y, y(1) = 0 \text{ find } y(1.2) \text{ with } h = 0.1.$$

OR

2. a) Given $\frac{dy}{dx} = y - x, y(0) = 2$. Find $y(0.1)$ and $y(0.2)$ by R. K. 4th order method, correct to four decimal places. **6**
- b) Apply Milne's predictor - corrector method to solve the diff^m equation. **6**
- $$\frac{dy}{dx} = -xy^2 \text{ at } x = 0.8 \text{ given that } y(0) = 2, y(0.2) = 1.923, y(0.4) = 1.724, y(0.6) = 1.471.$$
- c) Apply modified Euler's method to solve $\frac{dy}{dx} = e^x + xy, y(0) = 0$ to compute $y(0.1)$ & $y(0.2)$. **6**

3. a) Find $Z^{-1} \left[\frac{2z^2 - 5z}{(z-2)(z-3)} \right]$ **6**
- b) If $Z[f_n] = F(z)$ then $Z \left\{ \frac{f_n}{n+k} \right\} = z^k \int_z^\infty \frac{F(z)}{z^{k+1}} dz$ **6**

OR

4. a) Find $Z[a^n \cosh 3n]$ & $Z[a^n \sinh 3n]$. 6
- b) Solve by using Z-transform : 6
 $y_{n+3} - 3y_{n+2} + 3y_{n+1} - y_n = u(n)$,
 $y(0) = y(1) = y(2) = 0$
where $u(n)$ is unit step function.

5. a) Solve the equation by Frobenius method. 6
 $2x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - (x+1)y = 0$
- b) Prove that $J_{n+3} + J_{n+5} = \frac{2}{x}(n+4)J_{n+4}$ 6

OR

6. a) Express the polynomial into Legendre polynomial $f(x) = 4x^3 - 2x^2 - 3x + 8$ 6
- b) Prove that $\int P_n dx = \frac{1}{2n+1}[P_{n+1} - P_{n-1}] + c$ 6
7. a) Three urns contains 6 red, 4 black, 4 red 6 black, 5 red, 5 black balls respectively. One of the urns is selected at random and a ball is drawn from it. If the ball is red, find the probability that it is drawn from the first urn. 6
- b) Two dice are tossed. The random variable X is sum of two numbers. Which can come up. Find probability function, distribution fun and draw a graph of both. 6

OR

8. a) The density fun. of a random variable is given by $f(x) = \begin{cases} 2x & , 0 \leq x \leq 1 \\ 0 & , \text{otherwise} \end{cases}$ 6
find $P(X > 1/2)$, $P(X < 1/2)$ and $F(x)$.
- b) The probability fun. of two discrete random variables x and y is given by - 6
 $f(x, y) = \begin{cases} c(x^2 + y^2) & , 1 \leq x \leq 5, 0 \leq y \leq 4 \\ 0 & , \text{otherwise} \end{cases}$
find C, $P(x=3)$, $P(y=2)$, $P(x \geq 3, y \leq 3)$ Marginal probability for x and y.
9. a) The density fun. of random variables x & y is given by 7
 $f(x, y) = \begin{cases} cxy & , 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & , \text{otherwise} \end{cases}$
Find $E(X)$, $E(Y)$, $E(XY)$.

b) The density fun of random variable is given by

$$f(x) = \begin{cases} cx^2 & , 0 \leq x \leq 2 \\ 0 & , \text{otherwise} \end{cases}$$

Find moment generating function and first two moments about origin and about mean.

7

OR

10. a) Let X have the density function

$$f(x) = \begin{cases} \frac{3}{4}(1-x^2) & , -1 \leq x \leq 1 \\ 0 & , \text{otherwise} \end{cases}$$

find skewness and Kurtosis of X.

7

b) Find variance and standard deviation of a random variable X having probability fun.

7

x	1/2	-1/2
f(x)	1/2	1/2

11. a) Verify central limit theorem in case where x_1, x_2, \dots, x_n are independent and identically distributed as Poisson distribution.

6

b) Five fair coins are tossed simultaneously find the probability fun. of the random variable X = number of heads. And compute the probabilities of obtaining no heads, precisely 1 head, at least 1 head not more than 4 heads.

6

OR

12. a) The mean grade on a final examination was 72 and the standard deviation was 9. The top 10% of the students are to receive A's. What is the minimum grade a student must get in order to receive an A ?

6

b) If 3% of the electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs.

6

i) more than 5

ii) betⁿ 1 & 3

iii) at the most 2 bulbs will be defective.
