B.E. Fourth Semester (Mechanical Engineering) (C.B.S.)

Applied Mathematics - IV

P. Pages : 3 **NKT/KS/17/7283**

Time : Three Hours * 0 5 1 1 * Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.
10. Table for area under standard normal curve is permitted.

1. a) Using Regula Falsi method, find the root of the equation $xe^x = \cos x$ correct utp three places of decimal. **6**

b) Solve the system of equations by Gauss Seidal method **6**

$4x + 11y - z = 33$

$6x + 3y + 12z = 36$

$8x - 3y + 2z = 20$

OR

2. a) Find the root of the equation $x + \log_{10} x - 3.375 = 0$ correct upto three decimal places by Newton Raphson method. **6**

b) Solve by Crout's method. **6**

$5x + 2y + z = 12$

$x + 4y + 2z = 15$

$x + 2y + 5z = 20$

3. a) Use Euler's modified method to find y(0.2) given that $\frac{dy}{dx} = x^2 + y^2,\ y(0) = 1$. **7**

Take h = 0.1.

b) Determine the largest eigen value and the corresponding eigen vector of the matrix **7**

$$A = \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix}$$

OR

4. a) Use Runge-Kutta method to find approximate value of y for x = 0.2 when $\frac{dy}{dx} = xy + y^2$, given y(0) = 1, take h = 0.1 **7**

b) Given $2\frac{dy}{dx} = (1+x^2)y^2$ and y(0) = 1, y(0.1) = 1.06, y(0.2) = 1.12, y(0.3) = 1.21 evaluate y(0.4) by Milne's predictor corrector method. **7**

5. a) IF $Z\{f(n)\} = F(z)$, then prove that $Z\{f(n+k)\} = z^k\left[F(z) - \sum_{i=0}^{k-1} f(i)z^{-i}\right]$, $k > 0$ **6**

b) Find $Z^{-1}\left\{\frac{16z^3}{(4z-1)^2(z-1)}\right\}$ by residue method. **6**

OR

6. a) Solve the difference equation **6**
$y_{n+2} + 5y_{n+1} + 6y_n = 6^n$, $y_0 = 0$, $y_1 = 1$ by Z - transform method.

b) Find Z - transform of $\left[\frac{(n+1)(n+2)}{2!}a^n\right]$ **6**

7. a) Solve the differential equation **8**
$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$ by Frobenius method.

b) Show that :

i) $J_{½}(x) = \sqrt{\frac{2}{\pi x}} \cdot \sin x$ **3**

ii) $J_{-½}(x) = \sqrt{\frac{2}{\pi x}} \cdot \cos x$ **3**

OR

8. a) Express $f(x) = x^3 - 5x^2 + x + 2$ in terms of Legendre's polynomial. **6**

b) Prove that :

i) $(2n+1)\,x\,P_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$ **4**

ii) $nP_n(x) = xP'_n(x) - P'_{n-1}(x)$ **4**

9. a) A random variable x has density function $f(x) = \frac{c}{x^2+1}$, $-\infty < x < \infty$, **6**

Find :

i) Constant C

ii) $P\left(\frac{1}{3} \le x^2 \le 1\right)$

iii) Distribution function.

b) The joint probability function of two discrete random variables x and y is given by - **8**

$$f(x, y) = \begin{cases} c(2x+y) & , \quad x = 0, 1, 2 \; ; \; y = 0, 1, 2, 3 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find :

i) Constant C ii) $P(X \geq 1, Y \leq 2)$

iii) Marginal probability function of X & Y

iv) Are X and Y are independent

OR

10. a) **6**

Let $f(x, y) = \begin{cases} e^{-(x+y)} & , \quad x \geq 0, y \geq 0 \\ 0 & , \quad \text{otherwise} \end{cases}$

be the joint density function of X and Y,

Find :

i) Marginal density function of X and Y

ii) Conditional density function of X given Y.

b) **8**

$$\text{Let } X = \begin{cases} 1 & , \quad \text{Prob. } 1/6 \\ 2 & , \quad \text{Prob. } 1/3 \\ 3 & , \quad \text{Prob. } 1/2 \end{cases}$$

Find :

i) Mean ii) Variance

iii) Moment generating function iv) Characteristic function

11. a) A machine produces bolt which are 10% defective. Find the probability that in a random sample of 400 bolts produced by this machine : **7**

i) between 30 and 50

ii) at the most 30

iii) 55 or more of the bolts will be defective

b) A discrete random variable x is binomially distributed with mean 6 and variance 2 find probability that $5 \leq x \leq 7$. **3**

c) Let x be uniformly distributed in $-2 \leq x \leq 2$. Find : **4**

i) $P(X < 1)$ ii) $P\left(|x-1| \geq \frac{1}{2}\right)$

OR

12. a) Find the probability of getting between 2 head to 4 heads in 10 tosses of fair coin using **6**

i) binomial distribution

ii) the normal approximation to a binomial distribution.

b) Find : **8**

i) Mean ii) Variance

iii) Standard deviation

iv) Moment generating function for the uniform distribution

$$f(x) = \begin{cases} 1/(b-a) & , \quad a \leq x \leq b \\ 0 & , \quad \text{otherwise} \end{cases}$$

****