B.E.(Computer Science & Engineering (New) / Computer Technology)
Semester Third (C.B.S.)

Applied Mathematics

Paper - I

P. Pages : 3 **KNT/KW/16/7232/7237**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.

1. a) If $L\{f(t)\} = F(S)$ then prove that **6**

$L\{f'(t)\} = sL\{f(t)\} - f(0)$

and hence find $L\left\{\frac{d}{dt}\left(\frac{\sin t}{t}\right)\right\}$.

b) Find $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$ by Convolution Theorem. **6**

OR

2. a) Express $f(t) = \begin{cases} t-1; & 1<t<2 \\ 3-t; & 2<t<3 \end{cases}$ **6**

in terms of unit step function and find its Laplace transform.

b) Solve $\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t$, given y(0) = 1 by using Laplace Transform. **6**

3. a) Obtain Fourier Series for $f(x) = 1 + \frac{2x}{\pi}; -\pi \le x \le 0$ **6**

$= 1 - \frac{2x}{\pi}; 0 \le x \le \pi$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$

b) Solve the integral equation **6**

$$\int_0^\infty f(t)\cos\lambda t\,dt = \begin{cases} 1, & 0 \le \lambda < 1 \\ 2, & 1 \le \lambda < 2 \\ 0, & \lambda \ge 2 \end{cases}$$

OR

4. a) Find Fourier sine transform of $\frac{e^{-ax}}{x}$. **6**

b) Draw the graph of the function **6**

$$f(x) = \begin{cases} -1, & -2 \le x \le -1 \\ x \ , & -1 \le x \le 1 \\ 1 \ , & 1 \le x \le 2 \end{cases}$$

Discuss the symmetry and find the Fourier series for the function.

5. a) Prove that $Z\{n^p\} = -Z \frac{d}{dz} Z\{n^{p-1}\}$, p is a positive integer, hence find $Z\{n\}$. **6**

b) Prove that $\frac{1}{n!} * \frac{1}{n!} = \frac{2^n}{n!}$ where $*$ is a convolution operation. **6**

OR

6. a) Find inverse Z - transform of $\frac{Z^2 + Z}{(Z-1)(Z^2+1)}$. **6**

b) By using Z - transform solve the difference equation **6**

$y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, given $y_0 = y_1 = 0$.

7. a) If $u = y^3 - 3x^2 y$, show that u is harmonic function. Find V and analytic function. **7**

b) Evaluate $\int_C \frac{\cos \pi Z^2}{(Z-1)(Z-2)} dz$, where C is circle $|Z| = 3$. **7**

OR

8. a) Expand $f(Z) = \frac{Z^2 - 1}{(Z+2)(Z+3)}$ in the region **7**

i) $|Z| < 2$

ii) $2 < |Z| < 3$ and

iii) $|Z| > 3$

b) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta} d\theta$ by contour Integration. **7**

9. a) Find eigen vectors for the matrix **6**

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

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b) If $A = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$, verify $2\sin A = (\sin 2)A$ by Sylvester's theorem. **6**

c) Determine the largest eigen value and corresponding eigen vector of the matrix : **6**

$$A = \begin{bmatrix} -4 & -5 \\ 1 & 2 \end{bmatrix}$$

OR

10. a) Verify Cayley Hamilton's Theorem for $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}$ and hence find A^{-1}. **6**

b) Are the following vectors are linearly dependent? If so, find the relation between them **6**

$X_1 = [1,1,1,3]$, $X_2 = [1,2,3,4]$, $X_3 = [2,3,4,7]$

c) Solve by matrix method $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 10y = 0$ given $y(0) = 3$, $y'(0) = 15$. **6**

11. a) Each of three identical jewellary boxes has two drawers. In each drawer of the first box there is a gold watch. In each drawer of the second box there is a silver watch. In one drawer of the third box there is a gold watch while in other there is a silver watch. If we select a box at random, open one of the drawer and find it to contain a silver watch. What is the probability that the other drawers has gold watch. **6**

b) Let X be a random variable having density function **6**

$$f(x) = \begin{cases} cx & : 0 \le x \le 2 \\ 0 & : \text{otherwise} \end{cases}$$

find (i) the constant C, (ii) $P\left(\frac{1}{2} < x < \frac{3}{2}\right)$ and (iii) the distribution function.

OR

12. a) A random variable X has prob. density function **6**

$$f(x) = \begin{cases} \frac{1}{b-a} & , \ a \le x \le b \\ 0 & , \ \text{otherwise} \end{cases}$$

find (i) mean of X (ii) variance of X (iii) first two moments about origin.

b) In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. of the distribution. **6**

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