B.E. Fourth Semester (Computer Technology / Computer Science Engineering / Information Technology / Computer Engineering) (C.B.S.)

Discrete Mathematics & Graph Theory Paper – I

P. Pages : 4

Time : Three Hours

KNT/KW/16/7288/7293/7298/7303

* 0 1 5 6 *

Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Due credit will be given to neatness and adequate dimensions.
9. Use of non programmable calculator is permitted.

1. a) Prove that (i) $(A \cap B)' = A' \cup B'$ and $(A \cup B)' = A' \cap B'$. **6**

b) Construct the truth table for the following formula. **2**
$(\sim q \to p) \vee q$.

c) Write the negation of **2**
"If the sky is cloudy, then it rains and if it rains then the sky is cloudy."

OR

2. a) Test the validity of the following argument. **5**
"If I study, then I will not fail in Mathematics. If I do not play basket ball, then I will study. But I failed in Mathematics ------ therefore I played basket ball.

b) Show by Mathematical induction that $n^3 + 2n$ is divisible by 3. **3**

c) Write the contrapositive of **2**
"If x is a boss, then x is bad".

3. a) Let A= {1, 2, 3, 4} and R = {(1, 2), (2, 3), (3, 4)} be a relation on A. Find transitive closure of R and draw its digraph. **6**

b) Let A = {a, b, c} and P(A) be its power set. Let $\subseteq$ be the partial order relation on it. Draw Hasse diagram of $(P(A), \subseteq)$. **6**

c) Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$. **6**

OR

4. a) If $f : x \to y$ and $g : y \to z$ and both f and g are one-one and onto. **6**
Show that –

i) $g \circ f$ is one-one & onto ii) $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

b) Let A = {1, 2, 3, 4, 5, 6, 7} and R be a relation on A given by R = {(x, y) : x – y is divisible by 3}. Prove that R is an equivalence relation. **6**

c) Using properties of characteristic function prove that **6**

i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

ii) $\left(A^{c}\right)^{c} = A$.

5. a) Prove that fourth roots of unity forms an abelian group under multiplication. **6**

b) Show that the set of matrices $A_{\alpha} = \begin{bmatrix} \cos_{\alpha} & -\sin_{\alpha} \\ \sin_{\alpha} & \cos_{\alpha} \end{bmatrix}$, $\alpha \in R$ forms a monoid. **6**

OR

6. a) Define normal subgroup and prove that every subgroup of an abelian group is a normal subgroup. **6**

b) Show that the set $A = \{1, 2, 3\}$ under multiplication modulo 4 is not a group, but $B = \{1, 2, 3, 4\}$ under multiplication modulo 5 is a group. **6**

7. a) Let (L_1, D_6) and $L_2 = (P(S), S)$ be two lattices where $D_6 = \{1, 2, 3, 6\}$ and $S = \{a, b\}$. Then show that L_1 and L_2 are isomorphic. **6**

b) If R is a ring such that $a^2 = a \;\; \forall \; a \in R$, prove that **6**

i) a + a = 0

ii) $a + b = 0 \Rightarrow a = b$

iii) R is a commutative ring.

OR

8. a) Show that the set S of all matrices of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, $a, b \in R$ is a field with respect to matrix addition and multiplication. **6**

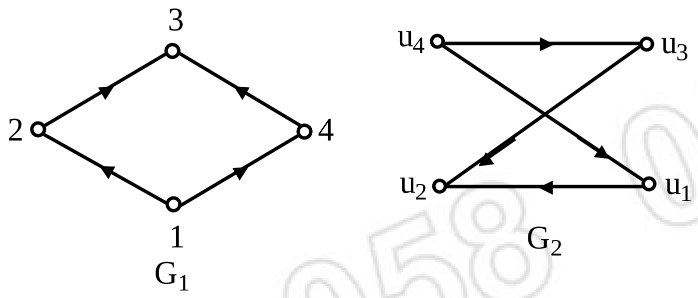
b) Construct switching circuit for the Boolean polynomial $(A \cdot B) + (A \cdot B') + (A' \cdot B')$. Simplify this and construct an equivalent circuit. **6**

9. a) Draw a digraph corresponding to the adjacency matrix **6**

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Give all possible paths of length 2. Is there any cycle of length 2?

b) Show that the two graphs G_1 and G_2 given below are isomorphic. **6**



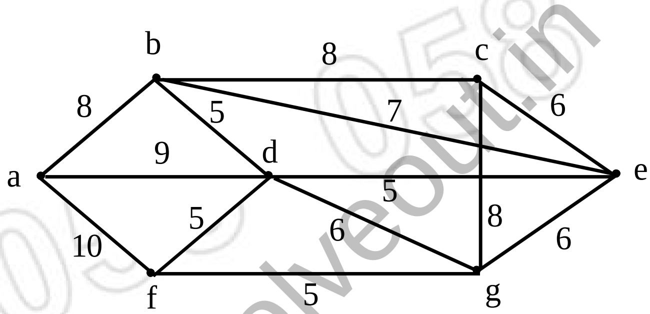
c) Draw tree and corresponding binary tree for the relation given by **6**

R = {(1, 2), (1, 3), (1, 4), (2, 5), (4, 6), (4, 7)} on set

A = {1, 2, 3, 4, 5, 6, 7}

OR

10. a) Apply prims algorithm to construct a minimal spanning tree for the weighted graph given below. **6**



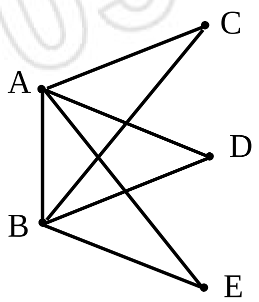
b) Define : **6**

i) Null graph ii) Trail

iii) Reachable node iv) Tree

v) Height of the tree vi) Radius of a graph

c) Define Eulerian path and Eulerian circuit. Show that the graph given below is an Eulerian graph and circuit. **6**



11. a) Solve the following recurrence relation by using generating function **5**

$a_n = 3a_{n-1} + 2,\ a_o = 1$

b) Show that if seven numbers from 1 to 12 are chosen then two of them will add upto 13. **5**

OR

12. a) Find the generating function of $n^2, n \geq 0$. **5**

b) Let n be a positive integer. Prove that **5**

$$\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+.....=\binom{n}{1}+\binom{n}{3}+\binom{n}{5}+.....$$

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