B.E. Fourth Semester (Electronics Engineering / Electronics Communication Engineering / Electronics Telecommunication Engineering) (C.B.S.)

Applied Mathematics - IV Paper - I

P. Pages : 3 **KNT/KW/16/7268/7273**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.
9. Use of normal probability table is permitted.

1. a) Find real root of the equation $xe^x - 3 = 0$ by method of false position. Correct upto four places of decimal. **6**

b) Solve the system of equations by Gauss Seidel method. **6**
$x + 7y - 3z = -22, \quad 5x - 2y + 3z = 18,$
$2x - y + 6z = 22.$

c) By Euler's Modified method, solve. **6**
$\frac{dy}{dx} = x + |\sqrt{y}|$, given $y = 1$, when $x = 0$
for $0 \le x \le 0.4$ taking $h = 0.2$

OR

2. a) By crout's method solve the system **6**
$2x + 3y + z = -1, \quad 5x + y + z = 9, \; 3x + 2y + 4z = 11.$

b) Solve $\frac{dy}{dx} = x + z, \frac{dz}{dx} = x - y^2$ for $x = 0.1$ given that $y(0) = 2, z(0) = 1,$ By Runge-Kutta method. **6**

c) Find larges Eigen value and corresponding Eigen vector for the matrix **6**
$$\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$
by Iteration process.

3. a) Prove that $Z\{n^P\} = -z\frac{d}{dz}Z\{n^{P-1}\},$ where P is any positive integer, using this find Z-transform of n, n^2, & n^3. **6**

b) Find inverse Z-transform of $\frac{z^2+z}{(z-1)(z^2+1)}$. **6**

OR

4. a) Find inverse Z-transform of $\frac{1}{(Z-2)^3}, |z|>2$. **5**

b) Solve the difference Equation $Y_{n+2}+6Y_{n+1}+9Y_n=2^n$, given $Y_0=Y_1=0$. **7**

5. a) Solve by frobenius method the equation $x\frac{d^2y}{dx^2}+\frac{dy}{dx}-y=0$. **6**

b) Express x^4-3x^2+x in term of Legendre's polynomials. **6**

OR

6. a) Prove that **6**

$xJ_n^{1}(x)=-nJ_n(x)+xJ_{n-1}(x)$.

b) Show that $\int_{-1}^{1} P_n(x)\,dx = \begin{cases} 0 & n\neq 0 \\ 2 & n=0 \end{cases}$ **6**

7. a) Three students A, B & C are in swimming race A and B have same probability of winning and each is twice as likely to win as C. Find the probability that B or C wins. **6**

b) Let X be the random variable giving the number of aces in a random draw of four cards from a pack of 52 cards. Find the probability function and the distribution function for X. **6**

OR

8. a) Prove for suitable constant C **6**

$F(x) = 0 \qquad x\le 0$

$= c\,(1-e^{-x})^2, \quad x>0$

is the distribution function. Also find density function and P (1< x <2).

b) Let X and Y be two random variables with joint probability function **6**

$f(x,y)=\frac{x+2y}{27}$, $x=0,1,2$ & $y=0,1,2$.

$=0 \qquad$ otherwise.

Find conditional probability function of y given X and X given Y.

9. a) A random variable X has density function given by **7**

$$f(x) = 2e^{-2x}, \quad x \geq 0$$
$$= 0 \quad x < 0$$

Find (i) the moment generating function.
(ii) first four moments about origin.

b) Let X and Y be random variables having joint density function **7**

$$f(x,y) = e^{-(x+y)}, \quad x \geq 0,\ y \geq o$$
$$= 0 \quad \text{otherwise}$$

Find i) var(x) ii) var(y) iii) σ_x
iv) σ_y v) σ_{xy} vi) ρ

OR

10. a) Let X and Y be random variables with joint density function. **7**

$$f(x,y) = x(1+3y^2), \quad 0 < x < 2,\ 0 < y < 1$$
$$= 0 \quad \text{otherwise}$$

Find marginal density function for X and Y & check if they are independent of each other or not.

b) The random variables X and Y having joint density function given by. **7**

$$f(x,y) = (2x+y)/210, \quad 2 < x < 6,\ 0 < y < 5$$
$$= 0 \quad \text{otherwise}$$

Find i) var(x) ii) var(y) iii) σ_x
iv) σ_y v) cov(x, y) vi) ρ

11. a) Verify central limit theorem for x_1, x_2 ------- x_n to be independent random variables, which are identically distributed with Poisson distribution. **6**

b) A machine produces bolts which are 10% defective. Find the probability that in a random sample of 400 bolts producted by this machine (i) between 30 and 50 (ii) at the most 30 (iii) 55 or more of the bolts will be defective. **6**

OR

12. a) In a photographic process, the developing time of prints may be looked upon as a random variable having the normal distribution with a mean of 16,28 seconds and a standard deviation of 0.12 seconds. Find the probability that it will take (i) anywhere from 16 to 16.50 sec to develop one of the prints (ii) at least 16.20 seconds to develop one of the prints (iii) at most 16.36 seconds to develop one of the print. **6**

b) In a certain factory turning razor blades, there is a probability of 1/500 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective (ii) two defective blades in a consignment of 10,000 packets. **6**

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