B.E.(Civil Engineering) Semester Third (C.B.S.)

Mathematics - III

P. Pages : 3 **KNT/KW/16/7207**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Due credit will be given to neatness and adequate dimensions.
9. Assume suitable data whenever necessary.
10. Use of non programmable calculator is permitted.

1. Find the Fourier series for the function. **7**

$$f(x)=\begin{cases}\pi+2x, & -\pi\le x\le 0\\ \pi-2x, & 0\le x\le \pi\end{cases}$$

Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+....+\infty=\frac{\pi^2}{8}$

OR

2. Find the half range sine series for the function. **7**

$$f(x)=\begin{cases}x, & 0\le x\le 2\\ 4-x, & 2\le x\le 4\end{cases}$$

3. a) Solve $(x^2-y^2-z^2)p+2xy\,q=2xz$ **6**

b) Solve $\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x\,\partial y}-8\frac{\partial^2 z}{\partial y^2}=e^{2x+y}+\sqrt{2x+3y}$. **6**

c) Solve $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+U$ given that $U(x,0)=6e^{-3x}$ by the method of separation of variables. **6**

OR

4. a) A tightly stretched string with fixed end points $x=0,\ x=\ell$ is initially at rest in its equilibrium position. It is set vibrating by giving each point a velocity $\lambda x(\ell-x)$, find the displacement of the string at any distance from one end at any time 't'. **8**

b) Solve $\frac{\partial^3 z}{\partial x^3}-3\frac{\partial^3 z}{\partial x\,\partial y}-2\frac{\partial^3 z}{\partial y^3}=\cos(x+2y)$. **5**

c) Solve : $x(y-z)p+y(z-x)q=z(x-y)$ **5**

5. Find the curve passing through the points (x_1, y_1) and (x_2, y_2) which when rotated about x - axis gives minimum surface area. **7**

OR

6. Find the extremals of the isoperimetric problem **7**

$$I = \int_0^{\pi} \left[(y^1)^2 - y^2\right] dx$$

given that $\int_0^{\pi} y dx = 1,\ y(0) = 0,\ y(\pi) = 1$.

7. a) Examine the following system of vectors for linearly dependent. Find the relation between them. $X_1 = (1,1,-1,1),\ X_2 = (1,-1,2,-1)\ X_3 = (3,1,0,1)$ **6**

b) Find eigen values, eigen vectors and model matrix for the matrix. **6**

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

c) Given $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, evaluate $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ by using Cayley - Hamilton theorem. **6**

OR

8. a) Use Sylvester's theorem to show that **6**
$3\tan[A] = (\tan 3)[A]$, where $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$.

b) Solve $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 10y = 0$, given $y(0) = 3,\ y'(0) = 15$ by matrix method. **6**

c) Reduce the quadratic form **6**
$6x^2 + 3y^2 + 3z^2 - 4xy + 4zx - 2yz$
to the canonical form.

9. a) Find the real root of the equation $xe^x = \cos x$ by Regula - Falsi method. **6**

b) Apply Gauss - Seidal method to solve the equations **6**
$2x - 3y + 20z = 25,\ 20x + y - 2z = 17$ and $3x + 20y - z = -18$.

c) Using modified Euler method, solve the differential equation **6**
$\frac{dy}{dx} = x + \sqrt{y}$ for $0 \le x \le 0.4$
given y = 1 when x = 0 and h = 0.2

OR

10. a) Use Runge - Kutta method to find approximate value of y for x = 0.2 when $\frac{dy}{dx} = x^2 + y^2$, given y (0) = 1, h = 0.1. **6**

b) Find a real root of the equation $x^3 - 2x - 5 = 0$ by Newton - Raphson method. **6**

c) Use Crout's method to solve the equations. **6**

$$4x + y - z = 13$$
$$3x + 5y + 2z = 21$$
$$2x + y + 6z = 14$$

11. A company manufactures three products A, B and C. Each product has to undergo operations on three types of machines m_1, m_2, m_3 before these are ready for sale. The time that each product requires on each machine are given in the following table. The table also show the net profit per unit on the sale of the three products. Formulate the mathematical model for this problem to maximize the total net profit of the company per day and obtain its solution by Simplex method. **12**

Machine ↓ Product →	Time required per unit (in minutes) A	B	C	Total time available per day (in minutes)
m_1	1	2	1	480
m_2	2	1	0	540
m_3	1	0	3	510
Profit Per Unit (Rs.)	4	3	5	

OR

12. a) Solve the following L.P.P. using Graphical method. **6**

Minimize $Z = 5x_1 + 8x_2$

Subject to $x_1 \le 4,\ x_2 \ge 2,\ x_1 + x_2 = 5$ and $x_1, x_2 \ge 0$

b) The standard weight of a special purpose brick is 5kg and it contains two basic ingredients B_1 and B_2. B_1 costs Rs. 5 per kg and B_2 costs Rs. 8 per kg. Strength considerations dictate that the brick contains not more than 4kg of B_1 and a minimum of 2kg of B_2. Since the demand for the product is likely to be related to the price of the brick. Formulate L.P.P. model to minimize the cost of the brick satisfying the above conditions. **6**

****