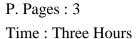
B.E. Fourth Semester (Mechanical Engineering) (C.B.S.)

Applied Mathematics - IV Paper – I





KNT/KW/16/7283

Max. Marks: 80

6

6

Notes: 1. All questions carry marks as indicated.

- 2. Solve Question 1 OR Questions No. 2.
- 3. Solve Question 3 OR Questions No. 4.
- 4. Solve Question 5 OR Questions No. 6.
- 5. Solve Question 7 OR Questions No. 8.
- 6. Solve Question 9 OR Questions No. 10.
- 7. Solve Question 11 OR Questions No. 12.
- 8. Assume suitable data whenever necessary.
- 9. Use of non programmable calculator is permitted.
- 10. Use of Normal Distribution table is permitted.
- 1. a) Find the root of equation $x^3 + x 1 = 0$ near to x = 1, by method of false position.
 - b) Apply Crout's method to solve the system of equations.

$$5x + 2y + z = 12$$

$$x + 4y + 2z = 15$$

$$x + 2y + 5z = 20$$

OR

- 2. a) Find by Newton Raphson method, the real root of $3x \cos x 1 = 0$, correct to four decimal places.
 - b) Solve 6x+15y+2z=72, 27x+6y-z=85, x+y+54z=110 by Gauss Seidel method.
- 3. a) Solve $\frac{dy}{dx} = 2e^x y$, given y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.040, y(0.3) = 2.090Find y(0.4) and y(0.5) by Milne's predictor corrector method.
 - Solve $\frac{dy}{dx} = 3x + y^2$, given y = 1.2 when x = 1. Find y(1.2) by Runge Kutta fourth order method, taking h = 0.1.

OR

4. a) Use modified Euler's method to solve equation $\frac{dy}{dx} = x + y$ for x = 0.1, given y(0) = 1, h = 0.05.

b) Find the largest Eigen value and corresponding Eigen vector for the matrix,

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

- 5. a) Prove that $Z\{n^p\} = -Z\frac{d}{dz}Z\{n^{p-1}\}$. Where P is any positive integer and hence find $Z\{n^2\}$ 6
 - b) Find inverse Z-transform of $\left[\frac{z^3}{(z-2)^3}\right]$, |z| > 2, using power series method.

OR

- 6. a) Find Z-transform of $\frac{(k+1)(k+2)}{2!}a^k$.
 - b) Solve $y_{n+2} + y_n = 2$, $y_0 = y_1 = 0$ by using Z-transform.
- 7. a) Solve in series by Frobenius method $2x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + 3y = 0$
 - b) Given $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$, $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ find $J_{\frac{3}{2}}(x)$ and $J_{-\frac{3}{2}}(x)$.

OR

- 8. a) Find $P_0(x), P_1(x), P_2(x), P_3(x)$ by using Rodrigue's formula.
 - b) If $f(x) = \begin{cases} 0, -1 < x < 0 \\ x, 0 < x < 1 \end{cases}$ then show that $f(x) = \frac{1}{4} P_0(x) + \frac{1}{2} P_1(x) + \frac{5}{16} P_2(x) \frac{3}{3^2} P_4(x) + \dots$

 $f(x) = \begin{cases} kx^2, & 1 \le x \le 2 \\ kx, & 2 < x < 3 \\ 0, & \text{otherwise} \end{cases}$

find the constant K and the distribution function.

b) The joint probability function of two discrete random variables X and Y is given by $f(x,y) = \begin{cases} cxy, & x = 1,2,3 \& y = 1,2,3 \\ 0, & \text{otherwise} \end{cases}$

Find:

- i) Constant C,
- ii) P(x = 3, y = 1)
- iii) $P(x \ge 2)$
- iv) find marginal probability function of x and y.

OR

10. a) Let X and Y be two random variables with joint probability function,

$$f(x,y) = \begin{cases} \frac{x+2y}{27}, & x = 0,1,2 & y = 0,1,2 \\ 0, & \text{otherwise} \end{cases}$$

Find conditional probability function of Y given X, and X given Y.

b) A random variable x has density function given by

$$f(x) = \begin{cases} 2e^{-2x}, & x \ge 0 \\ 0, & x < 0 \end{cases}$$

Find:

- i) E(x),
- ii) Var (x),
- iii) $E[(x-1)^2]$
- iv) the moment generating function.
- **11.** a) Define Exponential distribution and find its mean, variance and moment generating function.
 - b) If 3% of electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs (i) exactly 2, (ii) between 1 and 3 (iii) at most 2 bulbs will be defective.

OR

- 12. a) The mean grade on a final examination was 72 and the standard deviation was 9. The top 10% of the students are to receive A's. What is the minimum grade a student must get in order to receive an A?
 - b) Let x be uniformly distributed in $-2 \le x \le 2$ find:
 - i) P(x < 1)
 - ii) $P\left(|x-1| \ge \frac{1}{2}\right)$
 - c) The auto Correlation function for a stationary ergodic process is given by $R = (\tau) 25 + \frac{4}{3}$

$$R_{XX}(\tau) = 25 + \frac{4}{1 + \sigma \tau^2}$$

Find the mean and variance of the process x(t).

7

7