B.E. Fourth Semester (Mechanical Engineering) (C.B.S.)

Applied Mathematics - IV

Paper – I

P. Pages : 3 **KNT/KW/16/7283**

Time : Three Hours Max. Marks : 80

* 0 1 4 7 *

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.
10. Use of Normal Distribution table is permitted.

1. a) Find the root of equation $x^3 + x - 1 = 0$ near to $x = 1$, by method of false position. **6**

b) Apply Crout's method to solve the system of equations. **6**

$$5x + 2y + z = 12$$
$$x + 4y + 2z = 15$$
$$x + 2y + 5z = 20$$

OR

2. a) Find by Newton Raphson method, the real root of $3x - \cos x - 1 = 0$, correct to four decimal places. **6**

b) Solve $6x + 15y + 2z = 72,\quad 27x + 6y - z = 85,$ **6**

$x + y + 54z = 110$

by Gauss – Seidel method.

3. a) Solve $\frac{dy}{dx} = 2e^x - y$, given $y(0) = 2, y(0.1) = 2.010,\ y(0.2) = 2.040,\ y(0.3) = 2.090$ **7**

Find $y(0.4)$ and $y(0.5)$ by Milne's predictor corrector method.

b) Solve $\frac{dy}{dx} = 3x + y^2$, given $y = 1.2$ when $x = 1$. **7**

Find $y(1.2)$

by Runge Kutta fourth order method, taking $h = 0.1$.

OR

4. a) Use modified Euler's method to solve equation $\frac{dy}{dx} = x + y$ for $x = 0.1$, **7**

given $y(0) = 1,\ h = 0.05$.

b) Find the largest Eigen value and corresponding Eigen vector for the matrix, **7**

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

5. a) Prove that $Z\{n^p\} = -Z\frac{d}{dz}Z\{n^{p-1}\}$. Where P is any positive integer and hence find $Z\{n^2\}$ **6**

b) Find inverse Z-transform of $\left[\frac{z^3}{(z-2)^3}\right], |z| > 2$, using power series method. **6**

OR

6. a) Find Z-transform of $\frac{(k+1)(k+2)}{2!}a^k$. **6**

b) Solve $y_{n+2} + y_n = 2,\ y_0 = y_1 = 0$ by using Z-transform. **6**

7. a) Solve in series by Frobenius method **7**

$$2x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + 3y = 0$$

b) Given $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x,\ J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\cos x$ find $J_{3/2}(x)$ and $J_{-3/2}(x)$. **7**

OR

8. a) Find $P_0(x), P_1(x), P_2(x), P_3(x)$ by using Rodrigue's formula. **7**

b) If $f(x) = \begin{cases} 0, & -1 < x < 0 \\ x, & 0 < x < 1 \end{cases}$ then show that **7**

$$f(x) = \frac{1}{4}P_0(x) + \frac{1}{2}P_1(x) + \frac{5}{16}P_2(x) - \frac{3}{3^2}P_4(x) + \ldots$$

9. a) A random variable X has density function **6**

$$f(x) = \begin{cases} kx^2, & 1 \le x \le 2 \\ kx, & 2 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

find the constant K and the distribution function.

b) The joint probability function of two discrete random variables X and Y is given by **8**

$$f(x,y) = \begin{cases} cxy, & x = 1,2,3 \ \& \ y = 1,2,3 \\ 0, & \text{otherwise} \end{cases}$$

Find :

i) Constant C,

ii) $P(x = 3, y = 1)$

iii) $P(x \ge 2)$

iv) find marginal probability function of x and y.

OR

10. a) Let X and Y be two random variables with joint probability function, **7**

$$f(x,y)=\begin{cases}\frac{x+2y}{27}, & x=0,1,2\ \&\ y=0,1,2\\ 0, & \text{otherwise}\end{cases}$$

Find conditional probability function of Y given X, and X given Y.

b) A random variable x has density function given by **7**

$$f(x)=\begin{cases}2e^{-2x}, & x\ge 0\\ 0, & x<0\end{cases}$$

Find :

i) E(x),

ii) Var (x),

iii) $E\left[(x-1)^2\right]$,

iv) the moment generating function.

11. a) Define Exponential distribution and find its mean, variance and moment generating function. **7**

b) If 3% of electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs (i) exactly 2, (ii) between 1 and 3 (iii) at most 2 bulbs will be defective. **7**

OR

12. a) The mean grade on a final examination was 72 and the standard deviation was 9. The top 10% of the students are to receive A's. What is the minimum grade a student must get in order to receive an A? **6**

b) Let x be uniformly distributed in $-2\le x\le 2$ **4**
find :

i) $P(x<1)$

ii) $P\left(|x-1|\ge\frac{1}{2}\right)$

c) The auto Correlation function for a stationary ergodic process is given by **4**

$$R_{xx}(\tau)=25+\frac{4}{1+\sigma\tau^2}$$

Find the mean and variance of the process x(t).

****