B.E. All Branches Semester First (C.B.S.) / B.E. Semester First (Fire Engineering)

Applied Mathematics - I

Paper - I

P. Pages : 4 **KNT/KW/16/7196**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.

1. a) If $y = \cos(m \sin^{-1} x)$ **6**

prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$$

b) Evaluate **3**

i) $\lim\limits_{x \to 0} \left[\dfrac{1}{\sin^2 x} - \dfrac{1}{x^2} \right]$

ii) $\lim\limits_{x \to \frac{\pi}{2}} (\cos x)^{\cos x}$ **3**

OR

2. a) A curve is given by $x = a \sin\theta$; $y = b\cos 2\theta$. Find the radius of curvature at $\theta = \dfrac{\pi}{3}$. **6**

b) Expand $2x^3 + 7x^2 + x - 7$ in powers of $(x-2)$. **6**

3. a) If $u(x+y) = x^2 + y^2$ Then prove that **6**

$$\left[\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right]^2 = 4\left[1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right]$$

b) If $u = \tan^{-1}\left[\dfrac{x^{1/2} + y^{1/2}}{x^{1/3} - y^{1/3}}\right]$ **6**

find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$

c) If $u = f(2x-3y, 3y-4z, 4z-2x)$ **6**

Then prove that

$$\frac{1}{2}\frac{\partial u}{\partial x}+\frac{1}{3}\frac{\partial u}{\partial y}+\frac{1}{4}\frac{\partial u}{\partial z}=0$$

OR

4. a) If $u=\dfrac{x+y}{x-y}$ & $v=\dfrac{xy}{(x-y)^2}$ **6**

Then show that u & v are functionally related? If so find the relation between them.

b) Find the points on the surface $z^2 = xy+1$ nearest to the origin. **6**

c) Obtain Taylor's expansion of $\tan^{-1}\left(\dfrac{y}{x}\right)$ about (1, 1) upto the third degree terms. **6**

5. a) Find the inverse of matrix by partitioning **6**

$$A=\begin{bmatrix}1 & 2 & 3\\ 1 & 3 & 5\\ 1 & 4 & 12\end{bmatrix}$$

b) Test the consistency and solve **6**

$$5x+3y+7z=4$$
$$3x+26y+2z=9$$
$$7x+2y+10z=5$$

OR

6. a) Determine the rank of the matrix : **6**

$$A=\begin{bmatrix}1 & 2 & 3 & 0\\ 2 & 4 & 3 & 2\\ 3 & 2 & 1 & 3\\ 6 & 8 & 7 & 5\end{bmatrix}$$

b) Using adjoint method, solve **6**

$$x+2y+z=7$$
$$x+3z=11$$
$$2x-3y=1$$

7. a) $\dfrac{dy}{dx}-y\tan x = 3e^{-\sin x}$ **4**

b) $\dfrac{dx}{dy}=\dfrac{1+y^2+\cos^2 x}{y\sin 2x}$ **4**

c) $\frac{dy}{dx} + y \tan x = y^3 \sec x$

OR

8. a) $x^2\left(\frac{dy}{dx}\right)^2 + xy\frac{dy}{dx} - 6y^2 = 0$ **3**

b) Solve : $y + px = x^4 p^2$ **3**

c) The equation of electromotive force in terms of current i for an electrical circuit having resistance R and condenser of capacity C in series is : **6**

$E = Ri + \int \frac{i}{c} dt$ Find the current i at any time t when $E = E_m \sin wt$.

9. a) Solve $(D^2 - 5D + 6)\,y = e^{2x} + \cos x$. **6**

b) Solve by using variation of parameters method **6**

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = \frac{e^{2x}}{x}$$

c) Solve $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\log x$ **6**

OR

10. a) Solve $\frac{d^2y}{dx^2} = e^{-2y}$ **6**

Given that $y = 0, \frac{dy}{dx} = 0$ when $x = 0$.

b) Solve $\frac{d^2x}{dt^2} = b\frac{dy}{dt}$ and **6**

$$\frac{d^2y}{dt^2} = a - b\frac{dx}{dt}$$

c) A body executes damped forced vibrations given by the equation **6**

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + b^2x = e^{-kt}\sin wt$$

solve the equation when $w^2 = b^2 - k^2$.

11. a) If $2\cos\theta = x + \frac{1}{x}$ and **4**

$2\cos\phi = y + \frac{1}{y}$ then prove that

$$x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\theta + n\phi)$$

b) Find all the values of $\left[\frac{1}{2} + \frac{\sqrt{3}}{2} i\right]^{3/4}$ and show that the continued product of all the values is 1. **4**

OR

12. a) Solve using De - Moivre's theorem **4**

$$x^7 - x^4 + x^3 - 1 = 0$$

b) Separate $\tan^{-1}(x + iy)$ into real and imaginary parts. **4**

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