B.E. Second Semester (C.B.S.) / B.E. Second Semester (Fire Engineering)

Applied Mathematics - II Paper - II

P. Pages : 3 **KNT/KW/16/7202**

Time : Three Hours Max. Marks : 80

* 0 0 8 4 *

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

1. a) Evaluate $\int_0^1 \frac{x}{\sqrt{1-x^4}}\,dx$ **6**

b) By differentiation under the integral sign evaluate $\int_0^\infty \frac{e^{-ax}\sin x}{x}\,dx$ **6**

OR

2. a) Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta}\,d\theta$ **6**

b) A rod of length 'a' is divided into two parts at random. Prove that the mean value of the sum of squares on these two segments is $\frac{2}{3}a^2$. **6**

3. a) Trace the curve $a^2x^2 = y^3(2a-y)$ and show that its area is equal to πa^2. **6**

b) Find the perimeter of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$. **6**

OR

4. a) Find the volume of the solid obtained by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x-axis. **6**

b) Trace the cardioid r = a $(1+\cos\theta)$ and find the perimeter of the cardioid. **6**

5. a) Evaluate $\iint (x^2+y^2)\,dx\,dy$ over the region in the positive quadrant for which $x + y \le 1$. **6**

b) Evaluate $\int_0^a \int_y^a \frac{x^2}{\left(x^2+y^2\right)^{3/2}}$ dy dx by changing into polar form. **6**

c) Evaluate by changing the order of integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}$ dy dx **6**

OR

6. a) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz\, dz\, dy\, dx$ **6**

b) Find the mass of area bounded by the curves $y = x^2$ & $x = y^2$, if the density at any point is $\rho = \lambda\left(x^2+y^2\right)$. **6**

c) Evaluate $\iint \frac{r dr d\theta}{\sqrt{a^2+r^2}}$ over one loop of the lemniscate $r^2 = a^2 \cos 2\theta$. **6**

7. a) Show that. **6**

$\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)+\left(\vec{a}\times\vec{c}\right)\times\left(\vec{d}\times\vec{b}\right)+\left(\vec{a}\times\vec{d}\right)\times\left(\vec{b}\times\vec{c}\right)$ is parallel to the vector $\vec{a}$.

b) Find the directional derivative of $\phi(x,y,z) = x^2 - 2y^2 + 4z^2$ at the point (1, 1, -1) in the direction 2i + j - k. In what direction will the directional derivative be maximum and what is its magnitude? **6**

c) Prove that $\vec{A} = \left(6xy+z^3\right)\vec{i} + \left(3x^2-3\right)\vec{j} + \left(3xz^2-y\right)\vec{k}$ is irrotational. Find the scaler potential ϕ such that A = $\Delta\phi$. **6**

OR

8. a) A particle moves so that its position rector is given by $\vec{r} = \cos\omega t i + \sin\omega t j$ where ω is constant, prove that. **6**

i) Velocity $\vec{v}$ of the particle is perpendicular to $\vec{r}$.

ii) $\vec{r}\times\vec{v}$ = constant vector and.

iii) The acceleration $\vec{a}$ is directed towards the origin.

b) A particle moves along the curve $\bar{r} = \left(t^3-4t\right)i + \left(t^2+4t\right)j + \left(8t^2-3t^3\right)k$ where t is the time. Find the magnitude of the tangential and normal component of its acceleration at t = 2. **6**

c) Find the value of 'n' for which the vector field $r^n \vec{r}$ will be solenoidal. Find also whether the vector field $r^n \bar{r}$ is irrotational or not. **6**

9. If $\overline{A} = (y-2x)\, i + (3x+2y)\, j$, find the circulation of $\vec{A}$ about the circle C in the XY plane with Centre at origin and radius 2, C is traversed in the positive direction. **7**

OR

10. Use Green's theorem in the plane, evaluate $\int_c \left[\left(3x^2 - 8y^2\right) dx + (4y - 6xy)\, dy\right]$ Where C is the boundary of the region bounded by $y = \sqrt{x}$ and $y = x^2$. **7**

11. a) Fit a curve $y = ab^x$ to the following data. **7**

x	2	3	4	5	6
y	144	172.8	207.4	248.8	298.6

b) Find the function whose first order forward difference is $x^3 - 3x^2 + 9$. **6**

OR

12. a) In a partially distributed laboratory analysis of a correlation data, the following results only are eligible: **7**

$\sigma_x{}^2 = 9$

Regression equations: 8x - 10y + 66 = 0, 40x - 18y = 214 what were.

i) The mean values of x and y.

ii) Coefficient of correlation between x and y.

iii) Standard Deviation of y.

b) Solve the difference equation. **6**

$y_{n+2} - 2y_{n+1} + 4y_n = 2^n$

****