Faculty of Engineering & Technology

Third Semester B.E. (Electronics Engg.)/ET/EC/ Electrical/Mechanical (C.B.S.) Examination

APPLIED MATHEMATICS—III

Paper—III

Time : Three Hours] [Maximum Marks : 80

INSTRUCTIONS TO CANDIDATES

(1) All questions carry marks as indicated.

(2) Solve ***six*** questions as follows :

Question No. **1** **OR** Question No. **2**

Question No. **3** **OR** Question No. **4**

Question No. **5** **OR** Question No. **6**

Question No. **7** **OR** Question No. **8**

Question No. **9** **OR** Question No. **10**

Question No. **11** **OR** Question No. **12**.

(3) Use of non-programmable calculator is permitted.

1. (a) If $L\{f(t)\} = \bar{f}(s)$, then prove that :

$$L\left\{\int_0^t f(u)du\right\} = \frac{\bar{f}(s)}{s}.$$

Hence find $L\left\{\int_0^t \frac{\sin u}{u} du\right\}$. 7

(b) Express the function :

$$f(t) = \begin{cases} t^2, 0 < t < 2 \\ 4t, \quad t > 2 \end{cases}$$

in terms of unit step function and hence find Laplace transform. 5

OR

2. (a) Find $L^{-1}\left\{\frac{s^2}{(s^2+4)^2}\right\}$ using convolution theorem. 6

(b) Solve $\frac{dy}{dt} + 3y + 2\int_0^t y\, dt = t$, $y(0) = 0$ using Laplace transform method. 6

3. (a) Sketch the function :

$$f(x) = \begin{cases} 0, & -2 \le x \le -1 \\ 1+x, & -1 \le x \le 0 \\ 1-x, & 0 \le x \le 1 \\ 0, & 1 \le x \le 2 \end{cases}$$

and hence find Fourier series expansion of f(x). 6

(b) Using Fourier integral, prove that :

$$\int_0^\infty \frac{w \sin(xw)}{1+w^2} dw = \frac{p}{2} e^{-x}, x > 0$$ 6

OR

4. (a) Obtain half range fourier cosine series for $f(x) = \sin x$, $0 < x < \pi$. 6

(b) Solve the integral equation :

$$\int_0^\infty f(t) \cos ?\, t\, dt = \begin{cases} 1, & 0 \le ? < 1 \\ 2, & 1 \le ? < 2 \\ 0, & ? > 2 \end{cases}$$ 6

5. Find the plane closed curve of fixed perimeter and maximum area. 6

OR

6. Find the extremal of the functional :

$$\int_{x_0}^{x_1} \left\{x^2 (y')^2 + 2y^2 + 2xy\right\} dx$$ 6

7. (a) If u = $y^3 - 3x^2y$, show that u is harmonic. Also find v and corresponding analytic function f(z) = u + iv. 6

(b) Expand $f(z) = (z^2 + 4z + 3)^{-1}$ by Laurent's series valid for :

(i) $1 < |z| < 3$ and (ii) $|z| > 3$ 6

(c) Using contour integration, evaluate :

$$\int_0^{\infty} \frac{x \sin x}{x^2 + a^2} dx$$ 6

OR

8. (a) State Cauchy's integral formula and hence evaluate :

$$\oint_C \frac{\cos pz^2}{(z-1)(z-2)} dz, \text{ where } C : |z + i| = 1.5$$

6

(b) Evaluate :

$$\oint_C \frac{e^{zt}}{z(z^2+1)} dz, \; t > 2, \text{ where C is an}$$

ellipse $|z - \sqrt{5}| + |z + \sqrt{5}| = 6$. 7

(c) State Cauchy-Riemann conditions for the function f(z) to be analytic in the region R and test whether the function $f(z) = \log_e z$ is analytic. 5

9. (a) Solve the partial differential equation :

$$\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = (x + 2y)^{1/2} + e^{x+y}.$$

8

(b) Solve $yp - xq = -xe^{(x^2 + y^2)}$. 6

OR

10. (a) Solve $(D^2 + 3DD' + 2D'^2)z = 24\, xy$, where

$D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$. 7

(b) Using method of separation of variables,

$$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u,$$

given $u = 3e^{-y} - e^{-5y}$, when x = 0. 7

11. (a) Find whether the vectors :

$X_1 = [1\ 2\ 1]$, $X_2 = [2\ 1\ 4]$, $X_3 = [4\ 5\ 6]$ and $X_4 = [1\ 8\ -3]$ are linearly dependent. If so, find relation. 6

(b) Diagonalize the matrix :

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix}$$ 6

(c) Solve by matrix method :

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 0,$$

given $y(0) = 2$, $y'(0) = 5$. 6

OR

12. (a) If $A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$, find A^{10}. 6

(b) Verify Cayley-Hamilton theorem for the matrix

$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1}. 6

(c) Using Sylvester's theorem, show that :

$$\sec^2 A - \tan^2 A = I,$$

where $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$. 6