B.E. (Aeronautical Engineering) Third Semester (C.B.S.)

Applied Mathematics - III Paper - I

P. Pages : 3 **TKN/KS/16/7346**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Illustrate your answers whenever necessary with the help of neat sketches.
10. Use of non programmable calculator is permitted.

1. a) If $L\{f(t)\}=\bar{f}(s)$, then prove that $L\left\{\int_0^t f(u)\,du\right\}=\frac{\bar{f}(s)}{S}$. Hence find $L\left\{\int_0^t \frac{2(1-\cos u)}{u}\,du\right\}$. **6**

b) Express $f(t)=\begin{cases}\cos t, & 0<t<\pi\\ \sin t, & t>\pi\end{cases}$ in terms of unit step function and hence find its Laplace transform. **6**

OR

2. a) Solve the equation by using Laplace transform: $\frac{d^2x}{dt^2}+9x=\cos 2t$, given $x(0)=1,\ x\left(\frac{\pi}{2}\right)=-1$. **6**

b) Find $L^{-1}\left\{\frac{1}{\left(s^2+1\right)^2}\right\}$ by convolution theorem. **6**

3. a) Find the Fourier Transform of **6**

$f(x)=\begin{cases}1-x^2, & |x|<1\\ 0, & |x|>1\end{cases}$ and hence find $\int_0^\infty\left(\frac{\sin x-x\cos x}{x^3}\right)\cos\left(\frac{x}{2}\right)dx$.

OR

4. a) Solve the integral equation. **6**

$$\int_0^\infty f(x)\cos\lambda x\,dx=e^{-\lambda},\ \lambda>0.$$

5. a) Prove that $u = e^{-x}\{x\sin y - y\cos y\}$ is harmonic. Find V such that f (z) = u+iv is analytic and express f (z) in terms of z. **7**

b) Evaluate $\oint_C \frac{\cos \pi Z^2}{(Z-1)(Z-2)} dz$, where C is circle **7**

i) $|Z| = 3$ ii) $|Z+i| = 1.5$.

OR

6. a) Expand $f(z) = \frac{Z^2-1}{(Z+2)(Z+3)}$ in the regions. **7**

a) $|z| < 2$ b) $2<|z|<3$
c) $|z|>3$
by Laurent's series.

b) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$, by contour integration. **7**

7. a) Solve: P+3q=5z+tan (y-3x) where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. **6**

b) Solve: $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 3\frac{\partial^2 z}{\partial y^2} = \sqrt{x+3y}$. **6**

OR

8. a) Solve using method of separation of variables $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given that $u = 3e^{-y} - e^{-5y}$, when $x = 0$. **6**

b) A rod of length ℓ with insulated sides is initially at uniform temperature u_0. its ends are suddenly cooled to 0ºC and kept at that temperature. Find the temperature function u (x, t), if it satisfies the equation $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$. **6**

9. a) Are the following vectors Linearly dependent ? if so, find the relation between them. **6**
$x_1 = [2, -1, 3, 2], x_2 = [1, 3, 4, 2], x_3 = [3, -5, 2, 2]$.

b) Find the Modal Matrix B for the matrix. **6**

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}.$$

c) Verify Cayley-Hamilton theorem and express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial of A, where $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. **6**

OR

10. a) Use Sylvester's theorem to verify 2Sin A = (Sin 2) A, where $A=\begin{bmatrix}-1 & 3\\ 1 & 1\end{bmatrix}$. **6**

b) Solve : $\frac{d^2x}{dt^2}+4x=0$, given x(0) = 1, x'(0) = 0 by Matrix Method. **6**

c) Find the largest eigen value and corresponding eigen vector for the matrix. $\begin{bmatrix}-4 & -5\\ 1 & 2\end{bmatrix}$. **6**

11. a) Find the root of equation $x\log_{10}^{x}-1.2=0$ correct to third decimal place by using Regula-Falsi Method. **6**

b) Solve the following system of equations by Crout's Method. **6**

$$5x+2y+z=12,$$
$$x+4y+2z=15,$$
$$x+2y+5z=20$$

c) Solve the following system of equations by Gauss-Seidal Method. **6**

$$2x+10y+z=13$$
$$2x+2y+10z=14$$
$$10x+y+z=12$$

OR

12. a) Using Euler's modified method, solve the equation $\frac{dy}{dx}=x^2+y,\ y(0)=1,$ Find y(0.1) taking h = 0.05. **6**

b) Solve by Runge-Kutta fourth order method $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$, y (0) =1, find y (0.2) by taking h = 0.2. **6**

c) Solve $\frac{dy}{dx}=2e^x-y$, by Milne's predictor-corrector method given that $y(0)=2,\ y(0.1)=2.010,\ y(0.2)=2.040$, y (0.3) = 2.090, Find y (0.4). **6**

****