B.E. Second Semester Fire Engineering (C.B.S.)

Applied Mathematics - II Paper - I

P. Pages : 3 **TKN/KS/16/7290**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

1. a) Evaluate **6**

$$\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$$

b) Evaluate **6**

$$\int_0^{\infty} e^{-x^2} \cos \alpha x \, dx$$

by using the concept of differentiation under integral sign. Given

$$\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$$

OR

2. a) Prove that $\int_0^1 x^{n-1} \left[\log\left(1/x\right)\right]^{m-1} dx = \frac{\Gamma m}{n^m}$. **6**

b) Find the root mean square value of (a sin pt + b cos pt) over the interval 0 to 2π. **6**

3. a) Trace the curve $y^2 = x^2 - x^4$. **6**

b) Find the area lying between the parabola $y = 4x - x^2$ and the line y = x. **6**

OR

4. a) Find the volume of the solid generated by the rotation of the loop of the curve $y^2 = x^2 + x^3$ about the x-axis. **6**

b) Find the length of the curve $x = a\cos^3 t,\ y = a\sin^3 t$. **6**

5. a) Evaluate the double integral $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$ by changing it into polar coordinates. 6

b) Evaluate following integral by changing the order of integration $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\,dx$. 6

c) Evaluate $\iint y\,dx\,dy$, over the area bounded by $y = x^2$ and $x + y = 2$. 6

OR

6. a) Evaluate $\iint r\,dr\,d\theta$, over the area of the curve $r = a(1+\cos\theta)$ above the initial line. 6

b) Find the mass of the plate in the shape of the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$, the density being given by $\rho = \mu xy$. 6

c) Evaluate $\int_0^{\log_e 2} \int_0^x \int_0^{x+\log_e y} e^{(x+y+z)}\,dz\,dy\,dx$. 6

7. a) Prove that $[\bar{a} \times \{\bar{b} \times (\bar{c} \times \bar{d})\}] \cdot \bar{d} = (\bar{b} \cdot \bar{d})\{\bar{a} \cdot (\bar{c} \times \bar{d})\}$. 6

b) Find the directional derivative of $x^2 y^2 + y^2 z^2 + z^2 x^2$ at (1, 1, -2) in the direction of tangent to the curve $x = e^{-t}$, $y = 2\sin t + 1$, $z = t - \cos t$ at t = 0. 7

c) For what value of n, the vector field $\gamma^n \bar{\gamma}$ will be solenoidal? 5

OR

8. a) Find the tangential and normal components of acceleration at any time t, of a particle whose position at time t is given by $x = e^t \cos t$, $y = e^t \sin t$. 6

b) Show that $\bar{A} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is irrotational. Find the function ϕ such that $\bar{A} = \nabla\phi$. 6

c) Find the constants a and b so that the surface $ax^2 - 2byz = (a+4)x$ will be orthogonal to the surface $4x^2 y + z^3 = 4$ at the point (1, -1, 2). 6

9. Use Stoke's theorem to evaluate $\iint_S (\nabla \times \bar{F}) \cdot \hat{n}\,ds$, where $\bar{F} = yi + (x - 2xz)j - xyk$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the XY-Plane. 7

OR

10. A vector field is given by $\bar{F} = (2y+3)i + xzj + (yz - x)k$. Evaluate $\int_C \bar{F} \circ d\bar{r}$ along the path $x = 2t,\ \ y = t,\ \ z = t^3$ from t = 0 to t = 1. **7**

11. a) If y is the pull required to lift a load x by means of Pully block, find a linear law of the form y = mx + c connecting y and x using the following data: **7**

x	12	15	21	25
y	50	70	100	120

Also compute y when x = 150 kg.

b) Use Lagrange's interpolation to find y when x = 5 from the following data. **6**

x	0	1	3	8
y	1	3	13	123

OR

12. a) Find the coefficient of correlation and two lines of regression using following data: **7**

x	1	2	3	4	5
y	2	5	3	8	7

b) Solve the difference equation $y_{n+2} - 3y_{n+1} + 2y_n = 2n + 1 + 2^n$. **6**

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