VKR/KS/13/6443

Faculty of Engineering & Technology

Second Semester B.E. Examination

APPLIED MATHEMATICS—II

Paper—II

Time—Three Hours] [Maximum Marks—80

INSTRUCTIONS TO CANDIDATES

(1) All questions are compulsory.

(2) Solve **SIX** questions as follows :

Que. 1 **OR** 2, Que. 3 **OR** 4, Que. 5 **OR** 6,

Que. 7 **OR** 8, Que. 9 **OR** 10, Que. 11 **OR** 12.

(3) Use of non-programmable calculator is permitted.

(4) Assume suitable data wherever necessary.

1. (a) Evaluate :

$$\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta. \qquad 6$$

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(b) Evaluate $\int_0^1 \frac{x^a - 1}{\log x} dx$ by differentiating under integral sign, where $a > 0$. 6

OR

2. (a) Obtain the root mean square value of

$$f(t) = 3 \sin 2t + 4 \cos 2t$$

over the range $0 \leq t \leq \pi$. 6

(b) Prove that :

$$\int_0^\infty x^2 e^{-x^4} dx \times \int_0^\infty e^{-x^4} dx = \frac{\pi\sqrt{2}}{16}.$$ 6

3. (a) Trace the curve :

$$3ay^2 = x(x - a)^2.$$ 6

(b) Find the volume of the solid obtained by revolving the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

about X-axis. 6

OR

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4. (a) Find the area enclosed by the curve

$$r = a(1 + \cos\theta).$$ 6

(b) Find the length of arc of the parabola $y^2 = 4ax$ cut off by the latus rectum. 6

5. (a) Evaluate :

$$\int_0^1 \int_0^y xye^{-x^2}\, dydx$$ 5

(b) Evaluate :

$$\int_0^a \int_y^a \frac{x}{x^2 + y^2}\, dydx$$

by changing the order of integration. 7

(c) Evaluate :

$$\int_0^{\log 2} \int_0^x \int_0^{x + \log y} e^{(x+y+z)}\, dzdydx$$ 6

OR

6. (a) Evaluate :

$$\iint_R r^3\, drd\theta$$

over the area between the curves $r = 2\cos\theta$ and $r = 4\cos\theta$. 6

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(b) Find the mass of a plate in the shape of curve $\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=1$, the density being given by $\rho = \mu xy$. 6

(c) Evaluate :

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$$

by changing to polar co-ordinates. 6

7. (a) If $\bar{a}\times(\bar{b}\times\bar{c})=(\bar{a}\times\bar{b})\times\bar{c}$, then prove that :

$$(\bar{a}\times\bar{c})\times\bar{b}=\bar{0}.$$ 5

(b) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$, where t is the time. Find the components of its velocity and acceleration at $t = 1$ in the direction $i + j + 3k$. 6

(c) A vector field is given by :

$\bar{A}=(x^2+xy^2)i+(y^2+x^2y)j$. Show that the field is irrotational and find the scalar potential. 7

OR

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8. (a) Find the directional derivative of

$\phi(x, y, z) = x^2 - 2y^2 + 4z^2$ at the point $(1, 1, -1)$ in the direction of $2i + j - k$. In what direction will the directional derivative be maximum and what is its magnitude ?

6

(b) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.

6

(c) If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that :

(i) $\text{grad } r = \frac{\bar{r}}{r}$

(ii) $\text{grad}\left(\frac{1}{r}\right) = -\frac{\bar{r}}{r^3}$ and

(iii) $\nabla r^n = n r^{n-2} \bar{r}$

6

9. If $F = 2yi - zj + xk$, evaluate $\int_C F \times d\bar{r}$ along the curve

$x = \cos t$, $y = \sin t$, $z = 2 \cos t$ from $t = 0$ to $t = \frac{\pi}{2}$.

7

OR

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10. State Stoke's theorem, use it to evaluate :

$$\int_C \bar{F} \cdot d\bar{r}, \text{ where } \bar{F} = y^2 i + x^2 j - (x + z)k$$

and C is the boundary of the triangle with vertices (0, 0, 0), (1, 0, 0) and (1, 1, 0). 7

11. (a) Find the correlation coefficient and the equation of regression lines from the following data :

x	1	2	3	4	5
y	2	5	3	8	7

7

(b) Find the missing figure in the following table :

x	0	2	3	4	6
f(x)	−4	2	14	—	158

6

OR

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12. (a) Fit a parabola $y = a + bx^2$ for the following data by least square method : 7

x	1	2	3	4	5
y	1.8	5.1	8.9	14.1	19.8

1.186
0.738

(b) Solve :

$$y_{n+2} + 5y_{n+1} + 6y_n = n + 2^n.$$ 6

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