B.E. (Civil Engineering) Third Semester (C.B.S.)

# Mathematics - III

P. Pages : 3 **TKN/KS/16/7295**

Time : Three Hours Max. Marks : 80

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Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

**1.** Obtain Fourier Series for **7**

$f(x) = |\sin x|, -\pi < x < \pi$

Hence show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots \propto$$

**OR**

**2.** Obtain half range sine series for $f(x) = \pi x - x^2$ in the interval $(o, \pi)$. **7**

**3.** a) Solve : $(x^2 - yx)p + (y^2 - zx)q = z^2 - xy$ **6**

b) Solve : $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^2 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$. **6**

c) Solve using the method of separation of variables. **6**

$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given that

$u = 3e^{-y} - e^{-5y}$, when x = 0.

**OR**

**4.** a) A tightly stretched string with fixed end points x = o and $x = \ell$ is initially in a position given by $y = y_o \sin^3\left(\frac{\pi x}{\ell}\right)$. If it is released from rest from this position. Find the displacement y (x, t). **8**

b) Solve : $p + 3q = 5z + \tan(y - 3x)$. **5**

c) Solve : $\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x\,\partial y^2} + 6\frac{\partial^3 z}{\partial y^3} = \sin(x + 2y)$ **5**

**5.** Find the extremals of the functional $\int_1^2 \frac{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}{x} dx$ given y (1) = 0, y (2) = 1. **7**

**OR**

**6.** Find the plane closed curve of fixed perimeter and maximum area. **7**

**7.** a) Investigate the linear dependence of the vectors $X_1 = (3,1,-4)$, $X_2 = (2,2,-3)$, $X_3 = (0,-4,1)$, $X_4 = (-4,-4,6)$ and if possible find the relation between them. **6**

b) Find eigen values, eigen vectors and modal matrix for $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. **6**

c) Verify Cayley Hamilton theorem for matrix $A = \begin{bmatrix} 2, & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and find $A^{-1}$. **6**

**OR**

**8.** a) Diagonalise the following matrix by orthogonal transformation. **6**

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

b) Use Sylvester theorem to show that **6**

$$e^A = e^x \begin{bmatrix} \cosh x & \sinh x \\ \sinh x & \cosh x \end{bmatrix}$$

where $A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$

c) Solve $\frac{d^2y}{dx^2} + 4y = 0$ given y = 8, $\frac{dy}{dx} = 0$ when x = 0 **6**

**9.** a) Using the method of false position, find the root of the equation $x \log_{10}^{x} - 1.2 = 0$; correct upto three places of decimal. **5**

b) Apply Crout's method to solve the equations **6**

$$3x + 2y + 7z = 4$$
$$2x + 3y + z = 5$$
$$3x + 4y + z = 7$$

c) Given $\frac{dy}{dx} = x + y,\ y(0) = 1$, find y upto five terms by Picard's method and hence find y when x = 0.1 and x = 0.2 **7**

**OR**

**10.** a) Solve by 4th order Runge – Kutta method $\frac{dy}{dx} = xy + y^2$ given $y(0) = 1,\ h = 0.1$ find $y(0.1)$ and $y(0.2)$. **7**

b) Solve by Gauss Seidal Method. **6**

$$x + 7y - 3z = -22$$
$$5x - 2y + 3z = 18$$
$$2x - y + 6z = 22$$

c) Find by Newton Raphson method the root of the equation $e^x - 4x = 0$ near to 2, correct to three decimal places. **5**

**11.** A firm produces 3 products. These products are processed on 3 different machines. The time required to manufacture 1 unit of each of the 3 products and the daily capacity of the 3 machines are given in the following table. **12**

| Machine | Time per Unit | | | |
|---|---|---|---|---|
| | Product 1 | Product 2 | Product 3 | Machine capacity |
| M1 | 2 | 3 | 2 | 440 |
| M2 | 4 | – | 3 | 470 |
| M3 | 2 | 5 | – | 430 |

It is required to determine the daily number of units to be manufactured for each product. The profit per unit for product 1, 2 and 3 are Rs. 4, Rs. 3 and Rs. 6 respectively. It is assumed that the amounts produced are consumed in the market. Formulate and solve by Simplex Method.

**OR**

**12.** a) A farmer want to make sure that his herd get the minimum daily requirement of three basic nutrient A, B, C. Daily requirement are 15 unit of A 20 unit of B and 30 unit of C one gram of product P has 2 unit of A, 1 unit of B and 1 unit of C. One gram of product Q has 1 unit of A, 1 unit of B and 3 unit of C. The cost of P is Rs. 12/gram and cost at Q is Rs. 18/gram formulate this problem as linear programming problem so that the cost is minimum. **6**

b) Solve the linear programming using graphical method : **6**

Maximize $Z = 40x_1 + 60x_2$

Subject to : $4x_1 + 9x_2 \le 2000$

$12x_1 + 5x_2 \le 5000$

$6x_1 + 10x_2 \le 900$

$x_1, x_2 \ge 0$

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