

NTK/KW/15-7435

Fifth Semester B. E. (CT) (C.B.S.)  
Examination

DESIGN AND ANALYSIS OF ALGORITHM

Time : Three Hours ]

[ Max. Marks : 80

N. B. : (1) All questions carry marks as indicated.

(2) Solve Six questions as follows :

Que. No. 1 OR Que. No. 2

Que. No. 3 OR Que. No. 4

Que. No. 5 OR Que. No. 6

Que. No. 7 OR Que. No. 8

Que. No. 9 OR Que. No. 10

Que. No. 11 OR Que. No. 12

(3) Due credit will be given to neatness and adequate dimensions.

(4) Illustrate the answers with necessary figures/drawings wherever necessary.

1. (a) State and explain the Master's theorem for complexity analysis ? Also give its limitations. 6

(b) Derive a closed form solution for the summation :

$$S_k = \sum_{i=1}^{k-1} (i \cdot a^i) \quad 7$$

OR

2. (a) Solve the following recurrence relation with the help of characteristic equation method.

$$t_n = \begin{cases} 1 & \text{if } n=0 \\ 4t_{n-1} - 2^n & \text{otherwise} \end{cases} \quad 6$$

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Contd.

- ✓ (b) Solve following recurrence relation using change of variable method.

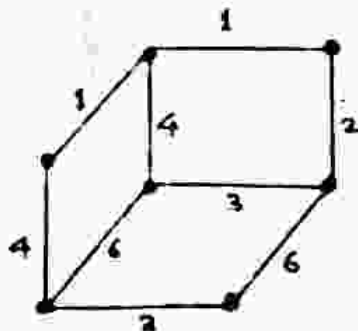
$$T(n) = 2T(\sqrt{n}) + \log n \quad 7$$

3. (a) State and explain in detail about various asymptotic notations which are used for analysis of algorithms. 7
- (b) Illustrate the stepwise execution of quicksort on the following array A. Also give its complexity by analyzing the recurrence relation.

$$A = [1, 3, 5, 8, 7, 6, 4] \quad 7$$

OR

4. (a) What do you mean by amortized analysis of algorithm? Explain any one method with suitable example. 7
- (b) Write an algorithm for binary search using divide and conquer strategy. Also give its stepwise execution for searching element  $X = 4$  in following input array.  
A [19, 3, 15, 8, 1, 6, 4] 7
5. (a) Find out Minimum spanning Tree with its cost for given undirected graph as follows. Use kruskal's algorithm.



6

- (b) Given 8 activities along with their start and finish time as follows :

| $A_i$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ | $A_8$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $S_i$ | 1     | 2     | 3     | 4     | 8     | 9     | 11    | 12    |
| $F_i$ | 5     | 2     | 4     | 7     | 11    | 12    | 13    | 16    |

Then compute a schedule where largest number of activities takes place using Greedy approach. 7

OR

6. (a) Find out optimal solution for fractional knapsack problem using Greedy strategy for following instances:

$$n = 7$$

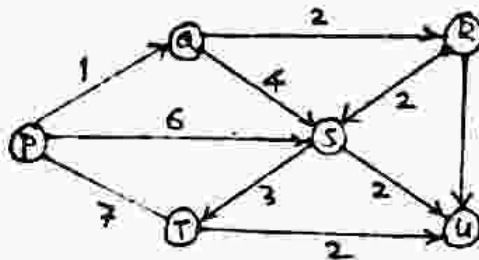
$$m = 15$$

$$(P_1, P_2, P_3, P_4, P_5, P_6, P_7) = (10, 5, 15, 7, 6, 18, 3)$$

$$(W_1, W_2, W_3, W_4, W_5, W_6, W_7) = (2, 3, 5, 7, 1, 4, 1)$$

6

- (b) Suppose we run Dijkstra's algorithm on edge directed graph with vertex P as source on following graph. Then in what order do nodes get included into set of vertices for which shortest distances are finalized.



7

7. (a) What is principle of optimality ? Explain in brief.

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(b) Determine the cost and structure of an optimal binary search tree for set of  $n = 5$  keys with following searching probabilities. Illustrate answer by dynamic programming.

| $i$   | 0    | 1    | 2    | 3    | 4    | 5    |
|-------|------|------|------|------|------|------|
| $p_i$ | -    | 0.15 | 0.10 | 0.05 | 0.10 | 0.20 |
| $q_i$ | 0.05 | 0.10 | 0.05 | 0.05 | 0.05 | 0.10 |

10

OR

8. (a) What is travelling salesman problem (TSP) ? Compute optimal TSP tour for following distance matrix using dynamic programming approach.

|   | A | B  | C  | D  |
|---|---|----|----|----|
| A | 0 | 10 | 15 | 20 |
| B | 5 | 0  | 9  | 10 |
| C | 6 | 13 | 0  | 12 |
| D | 8 | 8  | 9  | 0  |

7

(b) Solve the following instance of 0/1 knapsack problem using dynamic programming :

$$\text{Maximize : } x_1 + 2x_2 + 5x_3$$

subject to constraints :

$$2x_1 + 3x_2 + 4x_3 \leq 6$$

and restrictions,

$$0 \leq x_i \leq 1, 1 \leq i \leq 3$$

7

9. (a) Write and explain algorithms for iterative backtracking and recursive backtracking. 5
- (b) Consider  $S = \{S_1, S_2, S_3, S_4\}$  and weight vector  $W = \{10, 25, 5, 10\}$  and  $M = 25$ . Then compute all possible subsets of  $w$  that sum to  $m$ . Draw the portion of state-space tree that generates a fixed length tuple using backtracking algorithm. 8

**OR**

10. (a) What is  $M$ -colorability optimization problem in context of graph coloring? Explain the backtracking algorithm for graph coloring problem. 6
- (b) What is Hamiltonian cycle? Explain backtracking algorithm for it. 7
11. (a) How polynomial reduction can be used for showing NP-completeness of a problem? 7
- (b) Explain in detail about Cook's theorem. 6

**OR**

12. (a) What is NP-hard and NP-computer problem? Explain in detail. 6
- (b) What is clique? Comment about its NP-completeness. 7