B.E.Third Semester (Civil Engineering) (C.B.S.)

Mathematics - III

P. Pages : 3 **NKT/KS/17/7207**

Time : Three Hours * 0 2 1 9 * Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

1. Sketch the function **7**

$$f(x) = \begin{cases} \pi + x, & -\pi < x \le 0 \\ \pi - x, & 0 \le x < \pi \end{cases}$$

and find fourier series for f(x). Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + - - - - + \infty = \frac{\pi^2}{8}$

OR

2. Obtain half range cosine series for $f(x) = 2x - 1,\ 0 < x < 1.$ **7**

3. a) Solve $xq = yp + xe^{(x^2+y^2)}$ **6**

b) Solve $(D^2 - 3DD' + 2D'^2)\,Z = e^{2x+3y} + \sin(x - 2y)$ **6**

c) Solve by method of separation of variables the equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, given that $u(x,0) = 6e^{-3x}$. **6**

OR

4. a) A stretched string with fixed ends at $x = o,\ x = \ell$ is initially in a position given by $y(x, o) = a\sin\left(\frac{\pi x}{\ell}\right)$. If it is released from the rest, show that the displacement of any point at a distance x from one end at any time t is given by $y(x,t) = a\sin\left(\frac{\pi x}{\ell}\right)\cos\left(\frac{\pi x}{\ell}\right)$ **8**

b) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ **5**

c) Solve $(D^2 + DD' - 6D'^2)z = y\cos x$. **5**

5. Prove that the sphere is the solid of revolution which, for given surface area has maximum volume. **7**

OR

6. Find the extrenals of the functionals $\int_{x_0}^{x_1} \left[y^2 - (y')^2 - 2y\sin x\right] dx$. **7**

7. a) Show that the vectors $x_1 = [1, 0, 2, 1]$, $x_2 = [3, 1, 2, 1]$, $x_3 = [4, 6, 2, -4]$, $x_4 = [-6, 0, -3, -4]$ are linearly dependent. Find the relation. **6**

   b) Find the model matrix of $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ **6**

   c) Verify Cayley-Hamilton theorem and express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial of A, if $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. **6**

8. a) Diagonalise the matrix by orthogonal transformation. $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ **6**

   b) Use Sylvester's theorem to verify $\log_e e^A = A$ where $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$. **6**

   c) Solve $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 3y = 0$ given $y(0) = 2,\ y'(0) = 2$ by matrix method. **6**

9. a) Use Regula-false method to find the root of the equation $\tan x + \tanh x = 1$ correct to third decimal place. **5**

   b) Solve by Gauss - Seidel method **6**

$$2x + 10y + z = 13$$
$$2x + 2y + 10z = 14$$
$$10x + y + z = 12$$

   c) Solve by using modified Euler's method the equation $\frac{dy}{dx} = \log(x + y)$, given y(0) = 2 for x = 0.2 take h = 0.1. **7**

OR

10. a) Find by Newton-Raphson method the root of the equation $\sin x - \frac{x+1}{x-1} = 0$ near to $x = -0.4$. **5**

b) Solve by Runge-Kutta fourth order method $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$, find $y(0 \cdot 2)$ by taking $h = 0.2$. **6**

c) Solve the system of equations by Crout's method **7**

$$5x + 2y + z = 12$$
$$x + 4y + 2z = 15$$
$$x + 2y + 5z = 20$$

11. Solve by simplex method the L.P.P. **12**

Minimize : $z = 3x_1 - 7x_2 + 5x_3$

Subject to : $5x_1 - x_2 + 4x_3 \le 15$

$-3x_1 + 4x_2 \le 8$

$4x_1 + 3x_2 - 8x_3 \le 31$

OR

12. a) A farmer wants to make sure that his herd get the minimum daily requirement of three basic nutrients A, B, C. Daily requirements are 15 units of A, 20 units of B, 30 units of C. One gram of product P has 2 units of A, 1 unit of B and 1 unit of C. One gram of product Q has 1 unit of A, 1 unit of B & 3 units of C. The cost of P is Rs 12/gram and cost of Q is Rs. 18/gram. Formulate the L. P. P. to minimize the cost. **6**

b) Solve the L. P. P. by graphical method. **6**

Maximize : $z = 6x_1 + 4x_2$

Subject to : $2x_1 + 3x_2 \le 30$

$3x_1 + x_2 \le 24$

$x_1 + x_2 \ge 3$

$x_1, x_2 \ge 0$

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