Faculty of Engineering & Technology
Fourth Semester B.E. (Electronics Engineering/ET/EC) (C.B.S.) Examination
APPLIED MATHEMATICS—IV

Time—Three Hours] [Maximum Marks—80

INSTRUCTIONS TO CANDIDATES

(1) All questions carry marks as indicated.

(2) Solve **SIX** questions as follows :

Que. No. 1 **OR** Que. No. 2

Que. No. 3 **OR** Que. No. 4

Que. No. 5 **OR** Que. No. 6

Que. No. 7 **OR** Que. No. 8

Que. No. 9 **OR** Que. No. 10

Que. No. 11 **OR** Que. No. 12

(3) Use of normal probability table is permitted.

(4) Use of non-programmable calculator is permitted.

1. (a) Find a positive root of the equation $xe^x = 2$ by the method of False position. 6

(b) Solve the following system of equations by Crout's method :

$$4x + y - z = 13$$
$$3x + 5y + 2z = 21$$
$$2x + y + 6z = 14.$$ 6

(c) Use Runge-Kutta Fourth Order Method to find value of y for x = 0.2, when $\frac{dy}{dx} = \frac{x^2 + y^2}{10}$, y(0) = 1, h = 0.2. 6

OR

2. (a) Find a real root of the equation $x \log_{10} x - 1.2 = 0$ by Newton Raphson Method. 6

(b) Solve the following system of equations by Gauss Seidal Method :

$$2x + 10y + z = 13$$
$$2x + 2y + 10z = 14$$
$$10x + y + z = 12$$
6

(c) Solve :

$$\frac{dy}{dx} = 1 + xy^2, \; y(0) = 1 \text{ for } x = 0.4$$

by using Milne's Predictor-Corrector Method when it is given that

$$y(0.1) = 1.105, \; y(0.2) = 1.223, \; y(0.3) = 1.355$$
6

3. (a) If $Z\{f(n)\} = F(z)$, then prove that

$$Z\{f(n+k)\} = z^k \left[F(z) - \sum_{i=0}^{k-1} f(i) z^{-i}\right], k > 0$$

and hence find $Z\left\{\frac{1}{(n+1)!}\right\}$, given $Z\left\{\frac{1}{n!}\right\} = e^{1/z}$. 6

(b) By using Convolution theorem, find :

$$Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right].$$
6

OR

4. (a) Find Z-Transform of $\frac{(n+1)(n+2)}{2!} a^n$. 6

(b) Using Z-Transform method, solve

$$x_{n+2} + 3x_{n+1} + 2x_n = u_n,$$

given that $x_0 = 1$ and $x_n = 0$ for $n < 0$ where

$$u_n = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}.$$
6

5. (a) Solve in series the equation :

$$2x^2 \frac{d^2y}{dx^2} + (2x^2 - x)\frac{dy}{dx} + y = 0$$

by Frobenius Method. 6

(b) Prove that :

(i) $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \cdot \sin x$

(ii) $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cdot \cos x$. 6

OR

6. (a) If $f(x) = \begin{cases} 0, & -1 < x < 0 \\ x, & 0 < x < 1 \end{cases}$, then show that

$$f(x) = \frac{1}{4}P_0(x) + \frac{1}{2}P_1(x) + \frac{5}{16}P_2(x) - \frac{3}{32}P_4(x) + \dots$$ 6

(b) Show that $P_n(-x) = (-1)^n P_n(x)$ and hence prove that $P_n(-1) = (-1)^n$. 6

7. (a) A factory manufacturing televisions has four units A, B, C and D. The units manufacture 15%, 20%, 30% and 35% of the total outputs respectively. It was found that out of their outputs, 1%, 2%, 2% and 3% are defective. A television is chosen at random from the total output and was found to be defective. What is the probability that it was manufactured by unit C ? 6

(b) A random variable X has density function

$$f(x) = \frac{c}{x^2 + 1}, -\infty < x < \infty$$

Find :

(i) the constant C

(ii) $P\left(\frac{1}{3} \le x^2 \le 1\right)$

(iii) the distribution function. 6

OR

8. (a) Let $f(x, y) = \begin{cases} x + y, & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$

Find :

(i) Marginal density function of x and y

(ii) Marginal distribution function of x and y. 6

(b) The joint probability function of two discrete random variables x and y is given by :

$$f(x, y) = \begin{cases} cxy, & x = 1, 2, 3 \text{ and } y = 1, 2, 3 \\ 0, & \text{otherwise} \end{cases}$$

Find :

(i) the constant c

(ii) $P(x = 3, y = 1)$

(iii) $P(y < 2)$

(iv) Find marginal probability function of x and y. 6

9. (a) A coin weighted so that $P(H) = \frac{3}{4}$ and $P(T) = \frac{1}{4}$, is tossed three times. Let x be the random variable which denotes the longest string of heads which occurs. Find :

(i) Probability distribution

(ii) Expectation

(iii) Variance

(iv) Standard deviation of x. 7

(b) A random variable x has density function given by :

$$f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Find :

(i) E(x)

(ii) Var(x)

(iii) $E[(x-1)^2]$. 7

OR

10. (a) Let $f(x, y) = \begin{cases} 2e^{-(x+2y)}, & x \geq 0, y \geq 0 \\ 0, & x < 0, y < 0 \end{cases}$

Find :

(i) c

(ii) Conditional Expectation of y given x

(iii) Conditional Expectation of x given y. 7

(b) Find moment generating function and first four moments about the origin for random variable x given by :

$$x = \begin{cases} 1, & \text{Prob. } \frac{1}{2} \\ -1, & \text{Prob. } \frac{1}{2} \end{cases}$$ 7

11. (a) An insurance salesman sells policies to 5 men, all of identical age and in good health. The probability that a man of this particular age will be alive 30 years is $\frac{2}{3}$. Find the probability that in 30 years :

(i) All 5 men

(ii) At least 3 men

(iii) At most 1 man

(iv) At least 1 man

will be alive. 6

(b) In a certain factory turning out razor blades there is small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10; use Poisson distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 10,000 packets. 6

OR

12. (a) Verify central limit theorem in case where $x_1, x_2, \ldots, x_n$ are independent and identically distributed with Poisson distribution. 6

(b) In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of distribution. 6