III Sem. B.E. CIVIL (New Course)

WINTER - 2013

APPLIED MATHEMATICS - III

1. Obtain the Fourier series for the function : (7)

$$f(x) = 1 + \frac{2x}{\pi}, -\pi \le x \le 0$$

$$= 1 - \frac{2x}{\pi}, 0 \le x \le \pi$$

Deduce that :

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$$

(OR)

2. Expand $\pi x - x^2$ as half range Fourier sine series in the interval $(0, \pi)$. (7)

3. (a) Solve : $(x^2 - y^2 - z^2) p + 2xyq = 2xz$. (6)

(b) Solve : $\frac{\partial^2 z}{\partial x} - \frac{\partial^2 z}{\partial x \partial y} = \cos x \cos 2y$. (6)

(c) Solve using method of separation of variables $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ given that $u = 3e^{-y} - e^{-5y}$, when $x = 0$. (6)

(OR)

4. (a) Solve : $(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} + (mx - ly) = 0$. (6)

(b) Solve : $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \cos x$. (6)

(c) Use the method of separation of variables of solve wave equations $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$. (6)

5. Find the extremal of the functional :

$$V[y(x)] = \int_{x_0}^{x_1} [y^2 + y'^2 - 2y \sin x]dx$$ (7)

(OR)

6. Find the plane curve of fixed perimeter and maximum area. (7)

7. (a) Investigate the linear dependence of the vectors

$X_1 = (1, 2, 4), X_2 = (2, -1, 3), X_3 = (0, 1, 2),$

$X_4 = (-3, 7, 2)$ and if possible find the relation between them. (4)

(b) State Cayey Hamilton theorem and using it find A^{-1} where

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}.$$ (6)

(c) Reduce quadratic form, $2x^2 + 2y^2 - z^2 - 8xy + 4xz - 4yz$ to canonical form. Also specify matrix of transformation. (8)

(OR)

8. (a) Find characteristic equation eigen values and eigen vectors for

matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$ (6)

(b) Using Sylvester's theorem show that for any diagonal matrix

$M = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}, e^M = \begin{bmatrix} e^a & 0 \\ 0 & e^b \end{bmatrix}.$ (6)

(c) Solve $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} - 12x = 0$ given $x(0) = 0, x'(0) = 8$ by matrix method. (6)

9. (a) Use Network Raphson method to solve the equation : (6)

$3x - \cos x - 1 = 0$

(b) Solve the equations using Gauss Seidel method : (6)

$$x + 2y + 5z = 20$$
$$x + 4y + 2z = 15$$
$$5x + 2y + z = 12$$

(c) Using Milne's method find the solution of differential equation $\frac{dy}{dx} = x - y^2$ with $y(0) = 0$, $y(.2) = .020$, $y(.4) = .795$ and $y(.6) = .1762$ for $y(.8)$. (6)

(OR)

10. (a) Find the root of equation $x^3 - 3x + 4 = 0$ which lies between -2 and -3 using method of false position. (6)

(b) Solve the system of equations $x + y + z = 1$, $3x + y - 3z = 5$, $x - 2y - 5z = 10$ by Crout's method. (6)

(c) Use Runge Kulla method to find y when $x = 1.1$ given that $\frac{dy}{dx} = 3x - y^2$ and $y(1) = 1.5$. (6)

11. (a) Find all the basic solutions of the following system of equation identifying in each case the basic and non basic variables :(6)

$$2x_1 + x_2 + 4x_2 = 11,\ 3x_1 + x_2 + 5x_3 = 14.$$

Investigate whether the basic solutions are degenerate basic solutions or not. Hence find the basic feasible solution of the system.

(b) Using graphical method find the maximum value of $z = 2x + 3y$ subject to the constraint $x + y \le 30$, $y \ge 3$, $0 \le y \le 12$, $0 \le x \le 20$, $x - y \ge 0$ and $x, y, \ge 0$. (6)

(OR)

12. (a) One unit of product A contributes Rs. 7 and requires 3 units of raw material and 2 hours of labour. One unit of product B contributes Rs. 5 and requires one unit of raw material and one hour of labour. Availability of the raw material at present is 48 units and there are 40 hours of labour. (6)

(a) Formulate it as a linear programming problem. (6)

(b) Solve the above L.P.P. by Simplex method.

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