# B.E. Fourth Semester (Computer Technology) (C.B.S.) <br> Theory of Computation 

P. Pages: 3

NKT/KS/17/7291
Time : Three Hours

Notes : 1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Illustrate your answers whenever necessary with the help of neat sketches.

1. a) Define following properties on a relation.
1) Reflexive relation
2) Symmetric relation
3) Transitive relation
4) Equivalence relation
b) State and explain the pigeon hole principle.
c) Consider a relation $R=\{(a, b),(a, a),(b, b)$,

$$
(\mathrm{c}, \mathrm{c}),(\mathrm{d}, \mathrm{~d}),(\mathrm{b}, \mathrm{a})\}
$$

on a set $A=\{a, b, c, d\}$ Is relation $R$ a equivalence relation? If so, find the Equivalence classes.

## OR

2. a) Write a short note on Chomsky hierarchy.
b) Prove the following using method of Induction

$$
\sum_{i=1}^{n} i^{2}=\frac{n(n+1)(2 n+1)}{6}
$$

3. a) Convert the given non deterministic finite automata into a deterministic finite automata, State A is the initial state and states A, D are the final states.

| Present state | Next state |  |
| :---: | :---: | :---: |
|  | 0 | 1 |
| $\rightarrow \mathrm{~A}^{*}$ | $\{\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D})$ | - |
| B | $\{\mathrm{C}, \mathrm{D}\}$ | - |
| C | - | $\{\mathrm{D}\}$ |
| $\mathrm{D}^{*}$ | $\{\mathrm{D}\}$ | - |

b) Design a deterministic finite automata to accept the following strings over symbol 'a' and $\mathbf{2 x 3}$ 'b'
a) Strings that do not have 'bab' as a substring.
b) Strings that have 'aa' at most two times.
4. a) Minimise the given deterministic finite automata state A is the initial state and states $\mathrm{D}, \mathrm{F}$ are the final states.

| Present state | Next state |  |
| :---: | :---: | :---: |
|  | 0 | 1 |
| $\rightarrow \mathrm{~A}$ | B | D |
| B | A | D |
| C | B | E |
| $\mathrm{D}^{*}$ | F | F |
| E | D | D |
| $\mathrm{F}^{*}$ | F | F |

b) Differentiate between Moore machine and Mealy machine.
c) Design a Mealy machine to display the number of ' 0 's seen in the input string in mod 4 system. Input string may contain symbol '0' \& ' 1 '.
5. a) What is an ambiguous grammar? Show that the given grammar is an ambiguous grammar. $\mathrm{S} \rightarrow \mathrm{aSbS}|\mathrm{bSaS}| \in$
b) Convert the given context free grammar into a Chomsky normal form. Give also the definition of Chomsky normal form $\mathrm{S} \rightarrow \mathrm{abAB} ; \mathrm{A} \rightarrow \mathrm{bAB}|\in ; \mathrm{B} \rightarrow \mathrm{BAa}| \mathrm{A} \mid \in$
c) Give the statement of pumping lemma and show that a language $L=\left\{w w / w\right.$ is in $\left.(0 / 1)^{*}\right\}$ is a non regular language.

## OR

6. a) Design a regular grammar for the following language over symbol ' 0 ' and ' 1 '.
1) String should not start and end with same symbol.
2) String shuld have even number of 1's
b) Eliminate epsilon production from the given grammar
$\mathrm{S} \rightarrow \mathrm{aSB}|\mathrm{aA}| \mathrm{bB} ; \mathrm{A} \rightarrow \mathrm{aA}|\in ; \mathrm{B} \rightarrow \mathrm{bB}| \in$
c) Convert the given right linear grammar into it's equivalent left linear grammar.
$\mathrm{S} \rightarrow \mathrm{aaA}|\mathrm{bC} ; \mathrm{A} \rightarrow \mathrm{bbB}| \mathrm{b} ; \mathrm{B} \rightarrow \mathrm{cA} \mid \mathrm{c} ; \mathrm{c} \rightarrow \mathrm{d}$
7. a) Construct a push down automata for the following languages.
1) $\mathrm{L}=\left\{\mathrm{a}^{\left.\left.\mathrm{m}_{\mathrm{b}} \mathrm{c}^{\mathrm{p}} \mathrm{d}^{\mathrm{q}} \mid \text { where } \mathrm{m}+\mathrm{n}=\mathrm{p}+\mathrm{q}\right\},{ }^{2}\right\} .}\right.$
2) $\mathrm{L}=\left\{\mathrm{a}^{\mathrm{n}} \mathrm{b}^{\mathrm{m}} \mid\right.$ where $\left.\mathrm{n} \leq \mathrm{m}+3\right\}$
b) Show that the context free languages are closed under the property of closure and union.

## OR

8. a) State whether the following statements are true Justify your answer.
1) For every nondeterministic pushdown automata there exists a equivalent deterministic push down automata.
2) Push down automata is more powerful than finite automata.
3) Context free languages are closed under the property of Intersection.
b) Construct a push down automata for the following languages.
4) $\mathrm{L}=\left\{\mathrm{a}^{\mathrm{n}} \mathrm{b}^{\mathrm{m}} \mathrm{c}^{\mathrm{m}} \mid \mathrm{n} \geq 1\right.$ and $\left.\mathrm{m} \geq 1\right\}$
5) $\mathrm{L}=\left\{\mathrm{a}^{\mathrm{n}} \mathrm{b}^{2 \mathrm{n}} \mid \mathrm{n} \geq 1\right\}$
9. a) Give the formal definition of a Turing machine and linear bounded automata.
b) Write a short note on universal turing machine.
c) Design a turing machine to obtain 2's complement of a binary number.

## OR

10. a) Design a turing machine to accept a language
$L=\left\{a^{n} b^{n} c^{n} \mid\right.$ where $\left.n \geq 1\right\}$
b) List and explain the variants of a turing machine.
11. a) Show that $f(x, y)=x+y$ is a primitive recursive function.
b) Define the post correspondence problem and obtain solution for
1) $\mathrm{w}_{1}=11 ; \mathrm{w}_{2}=100 ; \mathrm{w}_{3}=111$
$\mathrm{v}_{1}=111, \mathrm{v}_{2}=001 ; \mathrm{v}_{3}=11$
2) $\mathrm{w}_{1}=00 ; \mathrm{w}_{2}=001 ; \mathrm{w}_{3}=1000$
$\mathrm{v}_{1}=0, \mathrm{v}_{2}=11 ; \mathrm{v}_{3}=011$
c) Give Ackermann's function and obtain solution for $\mathrm{A}(2,5)$ and $\mathrm{A}(3,3)$

## OR

12. a) Prove the following statements
1) If $L$ and $\bar{L}$ are both recursive enumerable language, then both $L$ and $\bar{L}$ are recursive
2) Union of two recursively enumerable languages is a recursive languages.
b) Write short note on undecidability
