B.E. Third Semester (Electronics / Electrical Engineering (Electronics & Power) / Electronics Tele Communication / Electronics Communication / Mechanical Engineering) (C.B.S.)

Applied Mathematics - III

P. Pages : 3 NKT/KS/17/7212/7217/7222/7227

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Attempt **six** questions as follows :
   Question 1 OR Questions No. 2.
   Question 3 OR Questions No. 4.
   Question 5 OR Questions No. 6.
   Question 7 OR Questions No. 8.
   Question 9 OR Questions No. 10.
   Question 11 OR Questions No. 12.
3. Use of non programmable calculator is permitted.

1. a) If $L\{f(t)\} = \overline{f}(s)$, then show that $L\left\{\frac{f(t)}{t}\right\} = \int_S^\infty \overline{f}(s)\,ds$; hence find $L\left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$. **6**

b) Find the inverse Laplace Transform of the function $\frac{1}{(S+1)(S^2+1)}$. **6**

OR

2. a) Express $f(t) = \begin{cases} \cos t \;, & 0 < t < \pi \\ \cos 2t \,, & \pi < t < 2\pi \\ \cos 3t \,, & t < 2\pi \end{cases}$ **6**

in terms of unit step function and hence find its Laplace transform.

b) Solve using Laplace transform method : **6**

$$\frac{d^2x}{dt^2} + 9x = \cos 2t; \quad x(0) = 1, \quad x\left(\frac{\pi}{2}\right) = -1$$

3. a) Expand $f(x) = x^2 \quad -\pi \le x \le \pi$ as a Fourier Series. Hence show that **6**

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots.$$

b) Express $f(x) = \begin{cases} 1, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}$ as a Fourier sine integral and hence evaluate **6**

$$\int_0^\infty \left(\frac{1-\cos \pi\lambda}{\lambda}\right) \sin x\lambda \, d\lambda \;.$$

OR

4. a) Find half range sine series of 6

$$f(x)=\begin{cases} x\,; & 0<x<\dfrac{\pi}{2} \\ \pi-x & \dfrac{\pi}{2}<x<\pi \end{cases}$$

b) Find the Fourier sine transform of $e^{-|x|}$ and hence show that 6

$$\int_0^\infty \frac{x\sin mx}{1+x^2}dx=\frac{\pi}{2}e^{-m}\,;\; m>0$$

5. Find the extremals of the functional 6

$$\int_{x_1}^{x_2}\left[16y^2-(y'')^2+x^2\right]dx$$

OR

6. Determine a function $y(x)$ such that the functional is extremum, given 6

$$\int_0^1 y^2dx=2;\quad y(0)=0=y(1)$$

where the functional

$$I=\int_0^1 (x^2+(y')^2)\,dx$$

7. a) If $u=y^3-3x^2y$ show that u is harmonic, find V and the corresponding analytic function. $f(Z)=u+iv$ 6

b) Evaluate $\oint_C \dfrac{\cos\pi Z^2}{(Z-1)(Z-2)}dZ$, where C is the circle $|Z|=3$. 6

c) Expand $\dfrac{1}{Z^2-3Z+2}$ in the region 6

i) $|Z|<1$ ii) $1<|Z|<2$ iii) $0<|Z-1|<1$

OR

8. a) Show that $u=e^{-x}(x\sin y-y\cos y)$ is harmonic. Find it's conjugate such that $f(Z)=u+iv$ is an analytic function. 6

b) State Residue theorem and use it to evaluate : 6

$$\oint_C \frac{12Z-7}{(Z-1)^2(2Z+3)}dZ,\text{ where C is the circle } |Z|=2$$

c) Using contour integration evaluate the integral : 6

$$\int_0^{2\pi}\frac{\cos 2\theta}{5+4\cos\theta}d\theta$$

9. a) Solve : **7**

$$x(y^2-z^2)p+y(z^2-x^2)q-z(x^2-y^2)=0$$

b) Solve : $\frac{\partial^3 z}{\partial x^3}-2\frac{\partial^3 z}{\partial x^2 \partial y}=2e^{2x}+3x^2y$ **7**

OR

10. a) Solve the Partial Differential Equation by the method of separation of variables $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$, given that $u=3e^{-y}-e^{-5y}$, where $x=0$. **7**

b) Using L. T. method, solve **7**

$$\frac{\partial u}{\partial t}+x\frac{\partial u}{\partial x}=x$$

where $x>0,\ t>0,\ u(x,0)=0\ \&\ u(0,t)=0$

11. a) Investigate the linear dependence of the vectors $X_1=(1,2,4),\ X_2=(2,-1,3),\ X_3=(0,1,2),\ X_4=(-3,7,2)$. If so, find the relations between them. **6**

b) Let $A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ find matrix P such that $\overline{P}^1AP$ is diagonal matrix. **6**

c) Given $A=\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$, show that $e^A=\begin{bmatrix} e^{\lambda_1} & 0 \\ 0 & e^{\lambda_2} \end{bmatrix}$ using Sylvester's theorem. **6**

OR

12. a) State Cayley Hamilton theorem. Verify it for the matrix A, where **6**

$A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ & hence find A^{-1}.

b) Solve $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+3y=0,\quad y(0)=2$ **6**

$$y'(0)=1$$

by matrix method.

c) Reduce the quadratic form $6x^2+3y^2+3z^2-4xy+4xz-2yz$ to canonical form where **6**

$$A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

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