B.E.(Electronics Engineering / Electrical Engineering (Electronics & Power) / Electronics Telecommunication / Electronics Communication / Mechanical Engineering) Semester Third (C.B.S.)

Applied Mathematics - III

Paper - I

P. Pages : 3 KNT/KW/16/7212/7217/7222/7227

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
   Attempt **six** questions as follows :
2. Question 1 OR Questions No. 2.
3. Question 3 OR Questions No. 4.
4. Question 5 OR Questions No. 6.
5. Question 7 OR Questions No. 8.
6. Question 9 OR Questions No. 10.
7. Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.

1. a) If $L\{f(t)\} = \bar{f}(s)$, then prove that **6**

$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} \bar{f}(s)$. Hence find $L[t \sin 2t]$.

b) Find $L^{-1}\left\{\frac{S^2}{(S^2+4)(S^2+9)}\right\}$ by convolution theorem. **6**

OR

2. a) If $f(t) = \cos 2t,\ \pi < t < 2\pi$ and **6**

$f(t) = 0$, otherwise,

express $f(t)$ in terms of unit step function and find its Laplace transform.

b) Solve $\frac{d^2x}{dt^2} + 9x = \cos(2t)$, **6**

given $x(0) = 1,\ x(\pi/2) = -1$, using Laplace transform technique.

3. a) Find Fourier Series for $f(x) = x - x^2$ in interval $-1 < x < 1$. **6**

b) Find Fourier Sine transform of $e^{-|x|}$ and hence show that : **6**

$$\int_0^\infty \frac{x \sin(mx)}{1+x^2} dx = \frac{\pi}{2} e^{-m},\ m > 0$$

OR

4. a) Obtain half range sine series for $f(x) = \pi x - x^2$ in the interval $(0, \pi)$. **6**

b) Find Fourier transform of **6**

$$f(x) = \begin{cases} 1, \text{ for } |x| < 1 \\ 0, \text{ for } |x| > 1, \end{cases}$$

hence find $\int_0^{\infty} \frac{\sin x}{x} dx$.

5. Find the extremals of **6**

$$V(y) = \int_{x_0}^{x_1} \left\{ (y'')^2 - 2(y')^2 + y^2 - 2y \sin x \right\} dx$$

OR

6. Find the curve passing through the points (x_1, y_1) and (x_2, y_2) which when rotated about x axis gives minimum surface area. **6**

7. a) Prove that $u = e^{-x}[x \sin y - y \cos y]$ is harmonic. Find V such that $f(z) = u + iv$ is analytic. **6**

b) If $f(a) = \oint_C \frac{3Z^2 + 7Z + 1}{Z - a} dz$, where C is a circle $|Z| = 2$, find values of **6**

i) $f(3)$

ii) $f'(1-i)$

iii) $f''(1-i)$

c) Expand the function $f(Z) = (Z^2 + 4Z + 3)^{-1}$ by Laurent'z Series valid for **6**

a) $|<|Z|<3$

b) $|Z| < 1$

c) $|Z| > 3$

OR

8. a) Find the value of $\oint_C \frac{(12Z - 7)}{(Z-1)^2 (2Z+3)} dz$ **6**

by using Residue theorem, where C is a circle $|Z| = 2$.

b) Evaluate $\int_0^{2\pi} \frac{1}{5 + 3\cos\theta} d\theta$ by contour integration. **6**

c) Evaluate $\int_0^{\infty} \frac{x \sin x}{x^2 + a^2} dx$ by contour Integration. **6**

9. a) Solve : 7

$$(mz-ny)p+(nx-\ell z)q=\ell y-mx$$

where $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$

b) Solve : 7

$$(D^2+2DD'-8D'^2)z=e^{2x+y}+\sqrt{2x+3y}$$

where $D=\frac{\partial}{\partial x}$, $D'=\frac{\partial}{\partial y}$

OR

10. a) Solve using method of separation of variables, 7

$4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$, given that

$u=3e^{-y}-e^{-5y}$ when $x=0$.

b) Solve using Laplace transform method : 7

$$\frac{\partial U}{\partial t}+x\frac{\partial U}{\partial x}=x,$$

$x>0,\ t>0,\ U(x,0)=0,\ U(0,t)=0$

11. a) Investigate the linear dependence of vectors 6

$X_1=[1,1,1,3],\ X_2=[1,2,3,4],\ X_3=[2,3,4,7]$

and if possible find the relation between them.

b) Find the Modal Matrix for the matrix 6

$$A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

c) Use Sylvester's theorem to show that 6

$2\sin A=(\sin 2)A$

where $A=\begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$

OR

12. a) If $A=\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, express $A^5-4A^4-7A^3+11A^2-A-10I$ as Linear polynomial of A. 6

b) Solve $\frac{d^2y}{dt^2}-3\frac{dy}{dt}-10y=0$, given $y(0)=3,\ y'(0)=15$ by Matrix Method. 6

c) Reduce the quadratic form $6x^2+3y^2+3z^2-4xy+4zx-2yz$ to the canonical form by an orthogonal transformation. 6

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