# Fourth Semester B.E. (Electrical Engineering) (C.B.S.) Examination

## APPLIED MATHEMATICS – IV

Time : Three Hours ]          [ Max. Marks : 80

> N. B. : (1) All questions carry marks as indicated.
> (2) Assume suitable data wherever necessary.
> (3) Use of non – programmable calculator is permitted.
> (4) Solve **six** questions as follows : Q. 1 or Q. 2 , Q. 3 or Q. 4 , Q. 5 or Q. 6 , Q. 7 or Q. 8 , Q. 9 or Q. 10 , Q. 11 or Q. 12.

1. (a) Define Transfer function. Obtain the mathematical model of the system given below. Also find transfer function of the system.
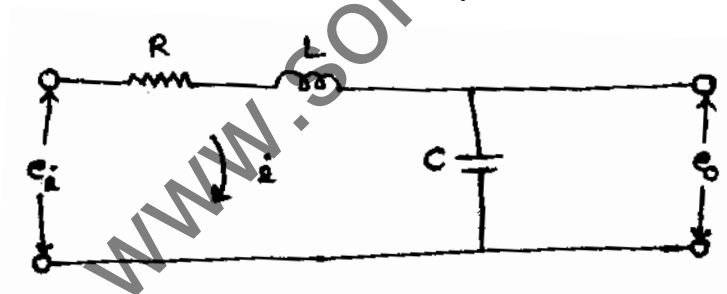


6

   (b) Define :—

   (i) Step signal

   (ii) Ramp Signal

   (iii) Parabolic signal and also find their Laplace Transform.                    6

**OR**

2. (a) Discuss the time response of a second order system for a unit step input. 6

   (b) Obtain unit step response of unity feedback system whose open loop transfer function is

   $$G(S) = \frac{4}{S(S+5)}$$ 6

3. (a) If $Z\{f(n)\} = F(z)$, show that

   $$Z\left\{\frac{f(n)}{n+k}\right\} = z^K \int_z^\infty \frac{F(z)}{Z^{K+1}} d_z$$

   Hence find $Z\left\{\dfrac{1}{n+1}\right\}$ 6

   (b) Find $Z$-transform of (a) $n$ and (b) $n^2$. 6

**OR**

4. (a) State convolution theorem and use it to find

   $$z^{-1}\left\{\frac{Z^2}{(z-a)(z-b)}\right\}.$$ 6

   (b) Solve $y_{n+2} + 5y_{n+1} + 6y_n = 6^n$, given $y_0 = 0, y_1 = 1$ by $Z$-transform. 6

5. (a) Define

   (i) Fuzzy set

   (ii) Normalized fuzzy set

   (iii) $\alpha$ – cut

   (iv) Support of fuzzy set. 6

(b) Let $\cup = \{\,a\,,b\,,c\,,d\,\}$ be the domain and A and B be fuzzy sets on $\cup$ as given below :

|   | a | b | c | d |
|---|---|---|---|---|
| A | 0.5 | 0.8 | 0.0 | 0.3 |
| B | 0.2 | 1.0 | 0.1 | 0.7 |

Find :—

  (i)   $A \cup B$

  (ii)  $A \cap B$

  (iii) A'

  (iv)  B'.

Also prove that (v) $(A \cup B)' = A' \cap B'$.     6

**OR**

6. (a) (i) Let $X = \{\,a\,,b\,,c\,\}$ and $Y = \{\,1\,,2\,,3\,\}$. The fuzzy sets A and B are defined on X and Y respectively as follows :—

$$A = \left\{\ \frac{0.2}{a}\ ,\frac{0.3}{b}\ ,\frac{0.5}{c}\ \right\}, B = \left\{\ \frac{0.5}{1}\ ,\ \frac{0.6}{2}\ ,\ \frac{0.7}{3}\ \right\}$$

Find $A \times B$.

(ii) If

| R | $y_1$ | $y_2$ | $y_3$ | $y_4$ |
|---|---|---|---|---|
| $x_1$ | 0.3 | 0.2 | 1 | 0 |
| $x_2$ | 0.8 | 1 | 0 | 0.2 |
| $x_3$ | 0.5 | 0.4 | 0.4 | 0 |

and

| S | $y_1$ | $y_2$ | $y_3$ | $y_4$ |
|---|---|---|---|---|
| $x_1$ | 0.3 | 0 | 0.7 | 0 |
| $x_2$ | 0.1 | 0.8 | 1 | 1 |
| $x_3$ | 0.6 | 0.9 | 0.3 | 0.1 |

Find   (i)   $R \cup S$   (ii)   $R \cap S$

$2 + 4 = 6$

(b) Let $X = \{ x_1, x_2, x_3 \}$ , $Y = \{ y_1, y_2, y_3, y_4, y_5 \}$ and $Z = \{ z_1, z_2, z_3, z_4 \}$. The fuzzy relations R and S are defined as follows :—

| R | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ |
|---|-----|-----|-----|-----|-----|
| $x_1$ | 0.1 | 0.2 | 0 | 0.7 | 0 |
| $x_2$ | 0.3 | 0.5 | 0 | 0.2 | 1 |
| $x_3$ | 0.8 | 0 | 1 | 0.4 | 0.3 |

and

| S | $z_1$ | $z_2$ | $z_3$ | $z_4$ |
|---|-----|-----|-----|-----|
| $y_1$ | 0.9 | 0 | 0.3 | 0.4 |
| $y_2$ | 0.2 | 1 | 0.8 | 0 |
| $y_3$ | 0.8 | 0 | 0.7 | 1 |
| $y_4$ | 0.4 | 0.2 | 0.3 | 0 |
| $y_5$ | 0 | 1 | 0 | 0.8 |

Find ROS. 6

7. (a) Find a real root of the equation $x^4 - 11x + 8 = 0$ accurate to four decimal places by the method of false position. 6

(b) Solve $10x + y + z = 12$

$2x + 10y + z = 13$

$2x + 2y + 10z = 14$ by Crout's method. 6

**OR**

8. (a) Find by Newton Raphson method, the root of the equation $\cos x = xe^x$. 6

(b) Solve $20x + y - 2z = 17$

$3x + 20y - z = -18$

$2x - 3y + 20z = 25$ by Gauss Siedal method. 6

9. (a) Solve $\dfrac{dy}{dx} = 2y + 3e^X$ , $y(0) = 0$ by Taylor's series method. Find $y(0.1)$ and $y(0.2)$. Compare the values with exact solution. 7

(b) Given

$$\frac{dy}{dx} = \frac{2 - y^2}{4x} \text{ and}$$

| x | 4 | 4.1 | 4.2 | 4.3 |
|---|---|------|------|------|
| y | 1 | 1.0049 | 1.0097 | 1.0144 |

Find $y(4.4)$ and $y(4.5)$ using Milen's Predictor Corrector Method. 7

**OR**

10. (a) Given that

$$\frac{dy}{dx} = 2 + \sqrt{xy} \text{ and } y = 1 \text{ when } x = 1$$

Find approximate value of y at $x = 1.6$ with $h = 0.2$ using Euler's modified method. 7

(b) Using Runge – Kutta method of order four, compute $y(0.3)$ from

$$10\frac{dy}{dx} = x^2 + y^2 , y(0) = 1. \text{ where } h = 0.3$$
7

11. (a) Urn A contains 1 white, 2 black , 3 red marbles. Urn B contains 2 white, 1 black , 1 red marbles. Urn C contains 4 white, 5 black , 3 red marbles. One Urn is choosen at random and two marbles are drawn. If one is white and other is red, then

what is the probability that they come from urn A ? 6

(b) Find the expectation of a discrete random variable, whose probability function is given by

$$f(x) = \left(\frac{1}{2}\right)^x, x = 1, 2, 3, .....$$

6

(c) The distribution function of a random variable X is given by

$$F(x) = \begin{cases} cx^3 & ; \ 0 \le x \le 3 \\ 1 & ; \ x \ge 3 \\ 0 & ; \ x < 0 \end{cases}$$

Find

   (i)   Constant C

   (ii)  Density function, f (x)

   (iii) P ( X > 1). 6

**OR**

12. (a) Find the moment generating function of a random variable X having density function

$$f(x) = \begin{cases} {}^{x}\!/_{2}, 0 \le x \le 2 \\ 0 \ , \text{otherwise} \end{cases}$$

Hence find first four moments about the origin. 6

(b) If 10% of bolts produced by a machine are defective. Determine the probability that out of 10 bolts chosen at random

   (i)   one

   (ii)  none

   (iii) at most '2' bolts will be defective. 6

(c) In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find.

   (i)   How many score between 12 and 15 ?

   (ii)  How many score above 18 ?

   (iii) How many score below 8 ?      6